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by

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ELECTRICAL POWER SYSTEM MODELS

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ABSTRACT

Analyses of electric power systems ordinarily include only a small portion of the entire network. Effects external to the network of interest are modeled as loads or generators. A method is proposed for identifying an equivalent model of the external network without taking any measurements in the external network itself. The method is simple and could easily be implemented to supplement state estimation techniques.

A linear load flow model which relates real power to voltage phase angle is developed for analyzing the entire network and from this two models are developed for relating measurements within the system of interest to the parameters of the equivalent model. It is found that there are two components to the equivalent model. One is a fixed structure and the other is an equivalent power. Since both models for identifying the equivalent network are in the form of an input, output relation with an unknown additive disturbance, a general solution for identifying linear static systems is found subject to certain conditions. Applying the solution to the two models for identifying the equivalent network, it is found that one of the models has an inherent tendency to produce consistently biased estimates of the equivalent model parameters. However, that same model has the advantage that it uses data for its input, output that is readily available and relatively accurate.

A simulation made to test the proposed method is discussed for a sample system and the results agree well in both the actual parameters identified and predicted error. Several important points are discussed with a view toward implementation.

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1. INTRODUCTION

Analyses of electrical power transmission networks usually carry the implication that the network under consideration can be isolated from all other networks. In actuality this can seldom be realized since the network is interconnected with one or more outside networks, and whatever happens to the network being analyzed is determined at least to some extent on how it interacts with the others. Therefore, it would be convenient if the effects of all the outside networks on the network of interest could be summarized by one equivalent network. In most cases the equivalent network would be considerably smaller than all the actual outside networks that it would represent. For instance, if the New England network were interconnected with the rest of the United States at a half dozen points, as far as the New England network would be concerned, the entire rest of the United States could be shrunk to an equivalent network having a half dozen buses. A method is proposed for finding the equivalent network which could be used for contingency planning and state estimation techniques [3], [4], [5], [8], [9], and [12].

1.1 General Problem: Identification of Linear, Static Systems

When analyzing a system, the usual approach is to surround the system of interest with a conceptual boundary. That within the boundary is idealized in some respect to emphasize certain features. Further, the idealized system is allowed to communicate with the outside only through certain conceptual ports. This idealized representation of actuality is called the system model and by analyzing the model conclusions can be drawn about the actual system. If the contents of the idealized system and what enters through the ports are completely known, then what happens to the idealized system can be found at least in principle. If the model is static, then only what currently enters is necessary to know the current state of the system. In such a case, if only knowledge of the current state is required, then the effect of the system outside the boundary is summarized by what enters through the ports. However, to know what will happen in the future, it is necessary to know what will enter in the future, and it may be that what enters through the ports is at least partly determined by how the system interacts with the outside. If everything outside the system is also idealized, then everything of interest can be modeled by the two idealized systems. The first, the original system model, represents what is within the boundary. The second, the external system model, represents all that is not represented by the first. Further, the two models can only interact through the originally specified ports. The external system model may be rather extensive and it might be desirable to find an equivalent external model which would interact with the original system model in the same way.

The process of finding the equivalent system model, called identification here, is investigated for a class of linear, static system models. More specifically, it is assumed that the model consists of a structure which remains constant and two types of variables, inputs (or causes), and outputs (or effects). The mathematical notation using matrices is simply $\underline{y} = \underline{C}\ \underline{u}$ where $\underline{u}$ represents an input vector, $\underline{y}$ represents an output vector, and $\underline{C}$ represents the structure.

If the original system model with its conceptual ports to the outside is placed in this framework, then it might be possible to separate what exists at or passes through the ports into the same two types of variables. For the model of a physical system, the product of the two variables frequently represents a generalized power passing through the ports. An example would be the voltage level existing at an electrical terminal and the current passing through the terminal. By combining inputs from the ports with inputs internal to the system, the outputs can be evaluated. In the same way knowledge of inputs at the ports, inputs within the external system model, and the structure of the external model allow outputs of the external model to be found. That is, each model may be analyzed independently of the other by knowing what passes through the ports. However, in order to predict future outputs of the original system, knowledge of how the two models interact is necessary. That is the purpose of identifying the equivalent external system model.

The inputs and outputs can be divided into those belonging to the original system, those of the external system, and those common to both. Two general approaches to identifying the equivalent model are investigated. One involves combining both the original and external models

and then identifying the new model using all the inputs not in the external model and only the outputs common to both. Another approach is to use the inputs and outputs at the ports or terminals to identify the equivalent external system separately. Both approaches are analogous to a black box which contains both the external model structure and inputs belonging to the external model. Inputs from the original system enter the black box and outputs are those which are common to both.

1.2 Specific Problem: Electrical Power Transmission Networks

Ordinarily when analyzing electrical power transmission systems, an arbitrary boundary is drawn about some part of the entire network and that part of the system is studied assuming power flowing in through the transmission lines on the boundary is known. The specific problem investigated is that of determining how an interconnected electrical power system affects a particular subsystem. The subsystem will be named "Own System" and referred to as OS for convenience. The rest will be named "External System" and referred to as XS. Together OS and XS form the entire system named "Whole System" or WS. The External System may be the entire outside world as viewed from one region. Or OS may be a higher voltage transmission network looking down into a more complex lower voltage distribution network. XS may also be a relatively extensive region where power and voltage measurements are not made which is surrounded by OS where measurements are made. To aid in analysis notation OS is further subdivided into "Internal System" or IS which has no immediate connections to XS and "Boundary System" which has immediate connections to XS. What is to be determined is an equivalent model for XS named "Equivalent System" or ES which will affect OS the same as XS.

It will be assumed that measurements are available in OS but not from XS. There are two aspects which set this particular identification problem apart from many others. One is that the identification must be primarily passive. It will not be possible to make any great manipulation of input signals (power flows) that unduly upset the system or do not conform to power consumption requirements. Also it can be expected

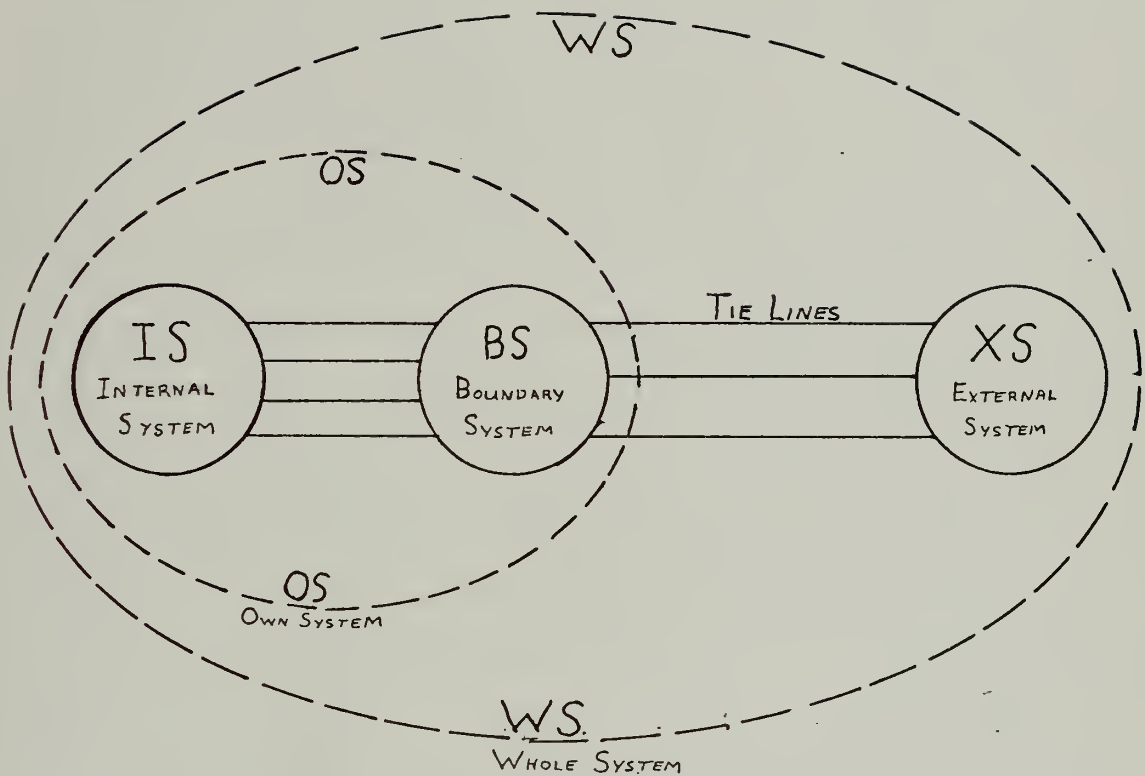


Figure 1.1 Separation of the Entire Network Into Regions

that power requirements in XS will be about the same as in OS, and since the unknown inputs in XS will cause unknown changes in the outputs of OS, "signal to noise ratios" will be about one to one.

1.3 Contents of Thesis

The basic conventional model which is used for analyzing networks will be developed in Section 2. Variables of interest for this model are real power, reactive power, voltage magnitude, and the relative voltage phase angle. The model is nonlinear and involves cross coupling of all variables. However it will be shown that under certain conditions the effect of voltage magnitude changes on real power is much less than the effect due to phase angle changes, so that an approximate linear model can be developed which relates real power to voltage phase angle. The linear model is not conventional but it is not new either. One of its chief advantages is a very significant reduction of computation requirements. But even though a linear model will be used for identifying the model for Equivalent System, it is still possible to incorporate the identified equivalent linear model in the nonlinear model for Own System. In Section 3 the linear model for External System is reduced to an equivalent model. Then two models for use in identifying Equivalent System are developed using the equivalent model. One of the models is attractive from the point of view that it only requires observations at the tie lines joining OS to XS, but it will turn out that there are inherent problems with this model due to the correlation among variables at the tie lines. Both models will be placed in a general form and in Section 4 the solution to the general problem will be found subject to certain conditions. The solution, which has a very simple form, is derived from both an assumed mathematical probability model for the disturbances and the method of weighted least squares. Error analysis equations are also derived to study the source of errors

and find a measure of confidence in the identified parameters. Applying the solution of the general problem to the two specific identification models in Section 5, the effects of the conditions placed on the solution are analyzed. One shortcoming of the more easily implemented model is discussed in terms of independent inputs. A simulation made to test the proposed methods of identification is discussed in Section 6, and the results show excellent correspondence with the theory. Errors in the identified parameters are within predicted accuracy. Finally, in Section 7 several aspects of the specific problem are discussed which involve increasing identification accuracy and verification of the identified model. It is also pointed out how models for identifying the equivalent system can be valuable by themselves for the purpose of network model reduction.

2. ELECTRICAL POWER TRANSMISSION MODELS

2.1 Nonlinear Load Flow Model

In reality an electrical power system consists of a large variety of devices, many capable of storing energy with time constants and natural frequencies spanning a very wide range. However for the purpose of computing power flows in sinusoidal steady state only a few types of components need be considered. The most important of these are the transmission lines which transmit power from points of generation to loads, the transformers, and the buses which form connecting points or nodes for transmission lines, transformers, generating stations, and loads. The values of interest are the power flows and bus voltages. Although the transformers, transmission lines, loads, and sources are capable of storing energy, when the system is in sinusoidal steady state, the usual approach is to treat energy storage devices as complex impedances or admittances and the sources and loads as injections of complex power. Hence although the system is oscillating, it can be analyzed as a static network.

A transmission line is of course a distributed parameter device. However as with most such devices, it is possible to model it as a set of lumped elements provided the physical size is small compared to the wavelength of the power transmitted. Since the wavelength of 60 hertz alternating current is roughly the distance between New York and San Francisco, while most transmission lines are far less, lumped element models are quite valid for this purpose. The pi model, common for this type of distributed parameter device, is used here and consists of a resistance and inductance in series with shunt capacitance at each end.

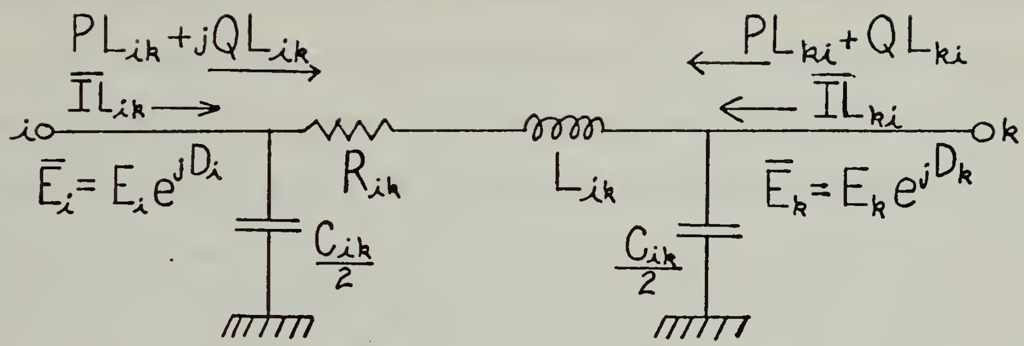


Figure 2.1 Pi Transmission Line Model

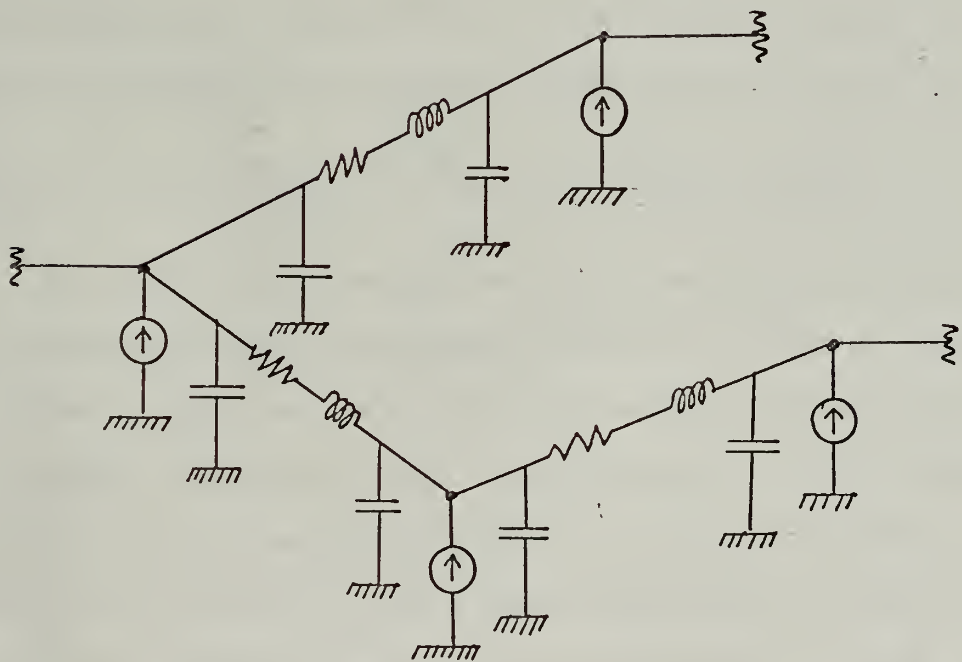


Figure 2.2 Electrical Power Network Lumped Model

Since the system studied is assumed to be in sinusoidal steady state, the capacitances and inductance can be represented as admittances. A bar above a symbol will denote a complex quantity (e.g. $\bar{E}_i = E_i e^{jD_i} = E_i \cos D_i + jE_i \sin D_i$). An underlined symbol will denote a matrix or vector. Let

$$\bar{y}_{ik} = y_{ik} e^{-j\phi_{ik}} = \frac{1}{R_{ik} + j\omega L_{ik}}$$

be the complex admittance of the series resistance and inductance, and let

$$\overline{ys}_{ik} = j\omega \frac{C_{ik}}{2}$$

be the admittance of the shunt capacitance. The current entering the transmission line at the i^{th} node is $\overline{IL}_i = \bar{E}_i \, \overline{ys}_{ik} + (\bar{E}_i - \bar{E}_k)\, \bar{y}_{ik}$. Complex power is $P + jQ = \bar{E}\,\bar{I}^*$ ($\bar{I}^*$ is the complex conjugate of $\bar{I}$) so the power entering the transmission line from the i^{th} node is

$$PL_{ik} - jQL_{ik} = \bar{E}_i^* \overline{IL}_{ik} = \bar{E}_i^* \bar{E}_i \, \overline{ys}_{ik} + \bar{E}_i^* (\bar{E}_i - \bar{E}_k)\, \bar{y}_{ik} \tag{2.1}$$

The network model consists of admittances which represent the transmission lines and transformers connected to buses or nodes. Also connected to each node is a current source which represents the load or generator at the bus. Power injections into the buses are positive for generators and negative for loads. As shown in Appendix A, a bus admittance matrix can be formed which represents the relation between the current sources at each bus to the bus voltages. Let $\underline{\bar{E}}_{bus}$ be a vector whose elements are the complex bus voltages, $\underline{\bar{I}}_{bus}$ a vector of the corresponding values of the complex bus current sources, and $\underline{\bar{Y}}_{bus}$

the complex bus admittance matrix. Then the relation is $\bar{I}_{bus} = \bar{Y}_{bus}\,\bar{E}_{bus}$ or for the i^{th} bus, $\bar{I}_i = \sum_{k=1}^{N} \bar{Y}_{ik}\bar{E}_k$ where there are N buses. Correspondingly, the complex power injected into the i^{th} bus by the current source at the bus is

$$P_i - jQ_i = \bar{E}_i^* \sum_{k=1}^{N} \bar{Y}_{ik}\bar{E}_k = E_i e^{-jD_i} \sum_{k=1}^{N} E_k e^{jD_k} Y_{ik} e^{-j\theta_{ik}}$$

$$= \sum_{k=1}^{N} E_i E_k Y_{ik} e^{-j(\theta_{ik}+D_i-D_k)} \qquad (2.2)$$

The load flow problem is that of given bus power injections, find the bus voltages and the transmission line power flows. It should be noted that even though the network is modeled with linear elements, the power flow equations are nonlinear.

2.2 Linear Real Power - Voltage Angle Model

Under certain assumptions it is possible to reduce the complexity of the model. From Equation (2.1) line real power flow is

$$PL_{ik} = \text{Real}\left\{jE_i^2 ys_{ik} + E_i^2 y_{ik} e^{-j\phi_{ik}} - E_i E_k y_{ik} e^{-j(\phi_{ik}+D_i-D_k)}\right\}$$

$$= E_i^2 y_{ik} \cos \phi_{ik} - E_i E_k y_{ik} \cos(\phi_{ik} + D_i - D_k)$$

$$PL_{ik} = E_i y_{ik} \cos \phi_{ik}\left[E_i - E_k \cos(D_i - D_k)\right] + E_i E_k y_{ik} \sin \phi_{ik} \sin(D_i - D_k) \tag{2.3}$$

If it can be assumed that voltage magnitudes do not vary much from nominal operating points, that the transmission line reactance is much greater than the resistance so that $\theta_{ik} \approx 90°$, and that voltage angle differences $(D_i - D_k)$ are small, then when $\tilde{E}_1 = (E_1)_{nominal}$, and $\tilde{E}_2 = (E_2)_{nominal}$, $PL_{ik} \approx \tilde{E}_1 \tilde{E}_2 y_{ik} \sin(D_i - D_k)$. For convenience it will be assumed that $\tilde{E}_1 = \tilde{E}_2 = 1.0$. Using the approximation for the sine, $\sin(D_i - D_k) \approx D_i - D_k$ for $D_i - D_k$ small

$$PL_{ik} \approx y_{ik}(D_i - D_k) \tag{2.4}$$

As shown in Appendix B_1 for such a case a linear load flow equation, $\underline{P}_{bus} = \underline{Y}_{bus}\underline{D}_{bus}$, relates real power injections to voltage angles. For a system with (K+1) buses, $\underline{P}_{bus}$ is a Kx1 vector of bus real power injections, $\underline{D}_{bus}$ is a Kx1 vector of bus voltage angles, and $\underline{Y}_{bus}$ is a KxK matrix whose elements are nearly the same as the magnitude of elements of the complex matrix $\underline{\bar{Y}}_{bus}$. The $(K+1)^{th}$ bus is the reference bus at which the voltage angle is specified. Power injection at the reference bus is such that the algebraic sum of real power injections of all buses is zero. Rom [6] and Rom and Schweppe [8] proposed the

use of this linear load flow model for on-line estimation of voltage angles and transmission line power flows while Baughman and Schweppe [2] showed how its results compare with the nonlinear model.

2.3 Discussion of the Linear Load Flow Model

Although the linear load flow model does not give the exact values of voltage angles obtainable by the nonlinear model nor does it account for voltage magnitude variations, there are some great advantages to using it for real power flow analyses even when the conditions on low line resistance and small voltage magnitude spread are only mildly met. Foremost is the simplicity of calculation. For (K+1) buses the nonlinear model requires an iterative solution of a set of 2K simultaneous nonlinear equations. The linear model involves K simultaneous linear equations. If the Newton-Raphson method is used to solve the first, some comparison between the two can be made. The Newton-Raphson method for one iteration is of the form $\underline{\Delta a}^m = \underline{B}^m \, \underline{\Delta c}^m$. Since calculations for solutions of simultaneous linear equations are about proportional to the square of the number of equations, one iteration for the nonlinear model requires about four times the calculations for the linear model. Also the $\underline{B}^m$ matrix must be calculated at each iteration whereas $\underline{Y}_{bus}$ in the linear model remains fixed for a given system. Finally, depending on the accuracy required, the Newton-Raphson method requires about four or more iterations. Another advantage is the ease with which the linear equations can be manipulated. This will become very important in transforming the network for finding an equivalent system as it results in much insight. Historically, before digital

computers were in wide use, load flow analyses were made with calculating boards which used physical components to model the network. The more versatile A-C calculating boards could account for voltage magnitudes and line power losses but were expensive and there were only about 50 in the United States [11]. The linear load flow model is somewhat analogous to the D-C calculating boards which were more numerous and less expensive. The linear load flow model accounts for individual bus real power injections while some D-C calculating boards used only one source.

The error involved in using the linear load flow model of course depends on how well the three conditions are met. The assumption of small voltage angle differences between the ends of a transmission line usually results in the least error. Except in rather rare circumstances, angle differences are within 30° and usually they are within 10°. Comparing the values of the sine of an angle and the angle:

Angle (degrees)	Angle (radians)	Sine
10°	$\frac{\pi}{18} = 0.1745$	0.1736
20°	$\frac{\pi}{9} = 0.349$	0.342
30°	$\frac{\pi}{6} = 0.524$	0.500

Even up to 30° the approximation is within 5%, and under 10° it is within 0.5%.

The error involved in the other two assumptions can be analyzed through the nonlinear equation for line power flow. If the line conductance, $g_{ik} = y_{ik} \cos \phi_{ik}$, and the line suseptance, $-b_{ik} = -y_{ik} \sin \phi_{ik}$, are used then equation (2.3) is

$$PL_{ik} = E_i g_{ik}[E_i - E_k \cos(D_i - D_k)] + E_i E_k b_{ik} \sin(D_i - D_k) \qquad (2.5)$$

In order to make a rather rough estimate of errors let "a" represent either b_{ik} or y_{ik}. Then when PL_{ik} is approximated by $b_{ik}(D_i - D_k)$ or $y_{ik}(D_i - D_k)$, for small angle differences

$$PL_{ik} \approx E_i g_{ik}(E_i - E_k) + E_i E_k b_{ik}(D_i - D_k)$$

and the relative error for $\widetilde{PL}_{ik} \approx a(D_i - D_k)$ is

$$\text{relative error} = \frac{\widetilde{PL} - PL}{PL} = \epsilon$$

$$\approx \frac{a(D_i - D_k) - E_i g_{ik}(E_i - E_k) - E_i E_k b_{ik}(D_i - D_k)}{E_i E_k b_{ik}(D_i - D_k)}$$

$$= \frac{a - E_i E_k b_{ik}}{E_i E_k b_{ik}} - \frac{1}{E_k} \frac{g_{ik}}{b_{ik}} \frac{E_i - E_k}{D_i - D_k}$$

For $E_i \approx 1.0$ and $E_k \approx 1.0$, ϵ is approximately

$$\epsilon \approx \frac{a}{b_{ik}} - E_i E_k - \frac{g_{ik}}{b_{ik}} \frac{E_i - E_k}{D_i - D_k}$$

If the voltage drop is expressed as a percentage, $\Delta E = 100(E_i - E_k)$, and if angle difference is expressed in degrees, $\Delta D = 57.3(D_i - D_k)$ $\approx 50(D_i - D_k)$, the relative error is approximately

$$\epsilon \approx \tilde{\epsilon} = \left(\frac{a}{b_{ik}} - E_i E_k\right) - \frac{1}{2} \frac{g_{ik}}{b_{ik}} \frac{\Delta E}{\Delta D} \qquad (2.6)$$

The purpose of equation (2.6) is to show how the greatest portion of the error enters the linear transmission line power flow equation (2.4). The term $\frac{a}{b_{ik}} - E_i E_k$ shows how voltage magnitude changes from nominal

values affect the error, while $\frac{1}{2}\frac{g_{ik}}{b_{ik}}\frac{\Delta E}{\Delta D}$ shows the effect of resistance and voltage drop. Table 2.1 compares line flows as computed both by the nonlinear and linear equations along with actual errors and approximate errors predicted by equation (2.6). It is not immediately obvious whether b_{ik} or y_{ik} (actually $\tilde{E}_i\tilde{E}_k b_{ik}$ or $\tilde{E}_i\tilde{E}_k y_{ik}$ where $\tilde{E}_i$ and $\tilde{E}_k$ are nominal values) should be used for a in $PL_{ik} \approx a(D_i - D_k)$.

In view of the relatively small error and much less involved calculations required, and to preserve insight, the linear load flow model is used for the purpose of identifying the equivalent system model.

E_i	E_k	g_{ik}	b_{ik}	D_i-D_k (degrees)	PL_{ik}	$\widehat{PL}_{ik} = y_{ik}(D_i-D_k)$			$\widehat{PL}_{ik} = b_{ik}(D_i-D_k)$		
							% Error			% Error	
						$\widehat{PL}_{ik}$	Actual	Estimated	$\widehat{PL}_{ik}$	Actual	Estimated
1.01	1.00	.10	1.0	5.0	.0894	.0877	-1.9	-1.5	.0873	-2.4	-2.0
1.00	1.01	.10	1.0	5.0	.0874	.0877	.3	.5	.0873	-0.2	0.0
1.05	1.00	.10	1.0	5.0	.0972	.0877	-9.7	-9.5	.0873	-10.2	-10.0
1.00	1.05	.10	1.0	5.0	.0869	.0877	0.9	0.5	.0873	0.4	0.0
1.01	1.00	.33	1.0	5.0	.0927	.0920	-0.7	1.1	.0873	-5.8	-4.3
1.01	1.00	.33	1.0	2.0	.0388	.0368	-5.2	-3.9	.0349	-10.1	-9.3
1.02	1.00	.20	1.0	5.0	.0937	.0890	-5.1	-4.0	.0873	-6.9	-6.0
1.02	1.00	.20	1.0	2.0	.0398	.0356	-10.6	-10.0	.0349	-12.3	-12.0

Table 2.1 Comparison of Line Power Flows Computed From the Linear and Nonlinear Models

3. MODELS FOR IDENTIFICATION OF THE EQUIVALENT SYSTEM

3.1 Tie Line Power Flow Model

As previously mentioned, two approaches to finding the equivalent system model will be used. The tie line power flow model views the external system as a separate system and attempts to find an equivalent model by using the tie line power flows and bus voltage angles where the boundary around OS crosses the tie lines connecting OS to XS. The bus injections and corresponding bus voltage angles can be separated into 3 groups, those belonging to IS (Internal System), those belonging to BS (Boundary System), and those belonging to XS (External System). Collectively IS and BS form OS (Own System). $\underline{PI}$ and $\underline{DI}$ are the bus real power injection vector and the bus voltage angle vector respectively whose elements belong to IS. Similarly $\underline{PB}$ and $\underline{DB}$ belong to BS, and $\underline{PX}$ and $\underline{DX}$ belong to XS. The letters I, B, and X will usually refer values belonging to IS, BS, and XS respectively. Since by earlier definition IS and XS are not immediately connected to each other by transmission lines, the linear load flow equations can be written:

$$\begin{bmatrix} \underline{PI} \\ \underline{PB} \\ \underline{PX} \end{bmatrix} = \begin{bmatrix} \underline{YII} & \underline{YIB} & \underline{0} \\ \underline{YIB}' & \underline{YBB} & \underline{YBX} \\ \underline{0} & \underline{YBX}' & \underline{YXX} \end{bmatrix} \begin{bmatrix} \underline{DI} \\ \underline{DB} \\ \underline{DX} \end{bmatrix} \tag{3.1}$$

(The transpose of a matrix will be denoted by $\underline{Y}'$ = transpose of $\underline{Y}$). The zero submatrices in the corners are due to the lack of direct coupling between IS and XS. The only coupling between OS and XS is reflected by the submatrix $\underline{YBB}$. Since the elements of $\underline{YBB}$ are values of admittances of transmission lines which have at least one of their

ends connected to buses in BS, $\underline{YBB}$ can be separated into that which belongs to OS, $\underline{YBOS}$, and that which belongs to XS, $\underline{YTL}$, so that $\underline{YBB} = \underline{YBOS} + \underline{YTL}$. $\underline{YTL}$ will be a diagonal matrix whose elements will be the admittances of the tie lines connecting OS to XS. Equation (3.1) can be written

$$\begin{bmatrix} \underline{PI} \\ \underline{PB} \\ \underline{PX} \end{bmatrix} = \begin{bmatrix} \underline{YII} & \underline{YIB} & \underline{0} \\ \underline{YIB}' & \underline{YBOS} & \underline{0} \\ \underline{0} & \underline{0} & \underline{0} \end{bmatrix} \begin{bmatrix} \underline{DI} \\ \underline{DB} \\ \underline{DX} \end{bmatrix} + \begin{bmatrix} \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & \underline{YTL} & \underline{YBX} \\ \underline{0} & \underline{YBX}' & \underline{YXX} \end{bmatrix} \begin{bmatrix} \underline{DI} \\ \underline{DB} \\ \underline{DX} \end{bmatrix}$$

or

$$\begin{bmatrix} \underline{PI} \\ \underline{PB} \end{bmatrix} = \begin{bmatrix} \underline{YII} & \underline{YIB} \\ \underline{YIB}' & \underline{YBOS} \end{bmatrix} \begin{bmatrix} \underline{DI} \\ \underline{DB} \end{bmatrix} + \begin{bmatrix} \underline{0} \\ \underline{YTL}\,\underline{DB} + \underline{YBX}\,\underline{DX} \end{bmatrix} \tag{3.2}$$

$$\underline{PX} = \underline{YBX}'\underline{DB} + \underline{YXX}\,\underline{DX} \tag{3.3}$$

Let $\underline{PTL} = \underline{YTL}\,\underline{DB} + \underline{YBX}\,\underline{DX}$ (3.4)

Tie line power flow, $\underline{PTL}$, is a vector of length equal to $\underline{PB}$ and represents the power which flows out of the buses belonging to BS into the transmission lines which connect BS to XS. Solving for $\underline{DX}$ in equation (3.3) and substituting into equation (3.4)

$$\underline{DX} = -\underline{YXX}^{-1}\underline{YBX}'\underline{DB} + \underline{YXX}^{-1}\underline{PX}$$

$$\underline{PTL} = \underline{YTL}\,\underline{DB} + \underline{YBX}(-\underline{YXX}^{-1}\underline{YBX}'\underline{DB} + \underline{YXX}^{-1}\underline{PX})$$

$$= (\underline{YTL} - \underline{YBX}\,\underline{YXX}^{-1}\underline{YBX}')\underline{DB} + \underline{YBX}\,\underline{YXX}^{-1}\underline{PX}$$

Define an equivalent external system bus admittance matrix, $\underline{YEQX}$, and an equivalent external bus power injection vector, $\underline{PEQX}$, such that

$$\underline{YEQX} = -\underline{YBX}\ \underline{YXX}^{-1}\underline{YBX}' \tag{3.5}$$

$$\underline{PEQX} = \underline{AQX}\ \underline{PX} \tag{3.6}$$

$$\underline{AQX} = -\underline{YBX}\ \underline{YXX}^{-1} \tag{3.7}$$

Then equations (3.2) and (3.4) can be rewritten

$$\begin{bmatrix} \underline{PI} \\ \\ \underline{PB} - \underline{PTL} \end{bmatrix} = \begin{bmatrix} \underline{YII} & \underline{YIB} \\ \\ \underline{YIB}' & \underline{YBCS} \end{bmatrix} \begin{bmatrix} \underline{DI} \\ \\ \underline{DB} \end{bmatrix} \tag{3.8}$$

$$\underline{PTL} = (\underline{YTL} + \underline{YEQX})\underline{DB} - \underline{PEQX} \tag{3.9}$$

Equation (3.8) is simply the linear load flow model for CS. By including PTL with PB the problem can be solved as usual. Equation (3.9) for PTL shows two contributions. The first, (YTL + YEQX)DB is the part of PTL due solely to the values of voltage angles existing at the buses of BS. The second, PEQX is the equivalent power injections in XS as seen by BS. It is the bus power injections from PX that are routed to BS by the structure of XS. It would be desirable to know both YEQX and PEQX, but even knowledge of just YEQX would be valuable. For example, if the voltage angles and line power flows in CS were wanted after a change in bus power injections in CS, then if PX (and thus PEQX) did not change between times t_1 and t_2, the change could be found by substituting equation (3.9) into (3.8) so that at any time t

$$\begin{bmatrix} \underline{PI}(t) \\ \\ \underline{PB}(t) + \underline{PEQX}(t) \end{bmatrix} = \begin{bmatrix} \underline{YII} & \underline{YIB} \\ \\ \underline{YIB}' & (\underline{YBB} + \underline{YEQX}) \end{bmatrix} \begin{bmatrix} \underline{DI}(t) \\ \\ \underline{DB}(t) \end{bmatrix}$$

and for the change with $\underline{PEQX}(t_2) = \underline{PEQX}(t_1)$

$$\begin{bmatrix} \underline{PI}(t_2) - \underline{PI}(t_1) \\ \\ \underline{PB}(t_2) - \underline{PB}(t_2) \end{bmatrix} = \begin{bmatrix} \underline{YII} & \underline{YIB} \\ \\ \underline{YIB}' & (\underline{YBB} + \underline{YEQX}) \end{bmatrix} \begin{bmatrix} \underline{DI}(t_2) - \underline{DI}(t_1) \\ \\ \underline{DB}(t_2) - \underline{DB}(t_1) \end{bmatrix}$$

where $\underline{DI}(t_1)$ and $\underline{DB}(t_1)$ are found by measuring $\underline{PTL}(t_1)$ and using equation (3.8). On the other hand, if the voltage angles and line power flows in OS were wanted after a change in the status of one or more transmission lines in OS, then if $\widetilde{\underline{YII}}$, $\widetilde{\underline{YIB}}$, and $\widetilde{\underline{YBB}}$ are the admittance matrices after the change, for $\underline{PI}(t_2) = \underline{PI}(t_1)$, $\underline{PB}(t_2) = \underline{PB}(t_1)$, and $\underline{PEQX}(t_2) = \underline{PEQX}(t_1)$

$$\begin{bmatrix} \underline{YII} & \underline{YIB} \\ \\ \underline{YIB}' & \underline{YBB}+\underline{YEQX} \end{bmatrix} \begin{bmatrix} \underline{DI}(t_1) \\ \\ \underline{DB}(t_1) \end{bmatrix} = \begin{bmatrix} \widetilde{\underline{YII}} & \widetilde{\underline{YIB}} \\ \\ \widetilde{\underline{YIB}}' & \widetilde{\underline{YBB}}+\underline{YEQX} \end{bmatrix} \begin{bmatrix} \underline{DI}(t_2) \\ \\ \underline{DB}(t_2) \end{bmatrix}$$

Everything is known except $\underline{DI}(t_2)$ and $\underline{DB}(t_2)$ since again $\underline{DI}(t_1)$ and $\underline{DB}(t_1)$ can be found by measuring $\underline{PTL}(t_1)$ and using equation (3.8).

As formulated, in equation (3.9) $\underline{DB}$ is an input vector and $\underline{PTL}$ is an output vector while $\underline{PEQX}$ might be thought of as an unknown disturbance. If a series of input, output measurements were taken, then it might be possible to find $\underline{YEQX}$. Equation (3.9) will be referred to as the tie line power flow model.

3.2 Boundary Bus Impedance Model

The second approach involves an impedance matrix instead of an admittance matrix. In order to obtain the required form, an equivalent internal system as seen by BS looking into IS will be found which is similar to the equivalent external system. The top row of equation (3.8) is

$$\underline{PI} = \underline{YII}\ \underline{DI} + \underline{YIB}\ \underline{DB}$$

Solving for $\underline{DI}$

$$\underline{DI} = -\underline{YII}^{-1}\underline{YIB}\ \underline{DB} + \underline{YII}^{-1}\underline{PI}$$

and substituting into the second row of equation (3.8)

$$\underline{PB} - \underline{PTL} = \underline{YIB}'(-\underline{YII}^{-1}\underline{YIB}\ \underline{DB} + \underline{YII}^{-1}\underline{PI}) + \underline{YBCS}\ \underline{DB}$$

Using equation (3.9) for $\underline{PTL}$ and combining terms

$$\underline{PB} + \underline{PEQX} - \underline{YIB}'\underline{YII}^{-1}\underline{PI} = (\underline{YBCS} + \underline{YTL} + \underline{YEQX} - \underline{YIB}'\underline{YII}^{-1}\underline{YIB})\underline{DB} \qquad (3.10)$$

Define an equivalent internal admittance matrix, $\underline{YEQI}$, and an equivalent internal bus power injection vector, $\underline{PEQI}$, similar to $\underline{YEQX}$ and $\underline{PEQX}$ by

$$\underline{YEQI} = -\underline{YIB}'\underline{YII}^{-1}\underline{YIB} \qquad (3.11)$$

$$\underline{PEQI} = \underline{AQI}\ \underline{PI} \qquad (3.12)$$

$$\underline{AQI} = -\underline{YIB}'\underline{YII}^{-1} \qquad (3.13)$$

Then equation (3.10) can be rewritten.

$$\underline{PEQI} + \underline{PB} + \underline{PEQX} = (\underline{YEQI} + \underline{YBB} + \underline{YEQX})\underline{DB} \qquad (3.14)$$

$\underline{PEQI}$ is the power injection of $\underline{PI}$ which is routed to the buses of BS by the structure of IS. $\underline{YEQI}$ is the structure of IS as seen by BS. $\underline{YBB}$ was previously separated into that belonging to OS, $\underline{YBOS}$ and that belonging to XS, $\underline{YTL}$. If $\underline{YBOS}$ is further separated into that belonging to IS, $\underline{YBIS}$ and that belonging to BS, $\underline{YBBS}$ so that $\underline{YBOS} = \underline{YBIS} + \underline{YBBS}$ or $\underline{YBB} = \underline{YBIS} + \underline{YBBS} + \underline{YTL}$, then the quantity

$$(\underline{YBIS} + \underline{YEQI})\underline{DB} - \underline{PEQI}$$

is the power flowing out of the buses of BS into the transmission lines connecting IS and BS. If BS had no transmission lines unique to itself, then $\underline{YBBS}$ would be zero. By rewriting equation (3.14) with $\underline{YBB} = \underline{YBIS} + \underline{YBBS} + \underline{YTL}$

$$\underline{PEQI} + \underline{PB} + \underline{PEQX} = (\underline{YBIS} + \underline{YEQI})\underline{DB} + \underline{YBBS}\ \underline{DB} + (\underline{YTL} + \underline{YEQX})\underline{DB} \qquad (3.14a)$$

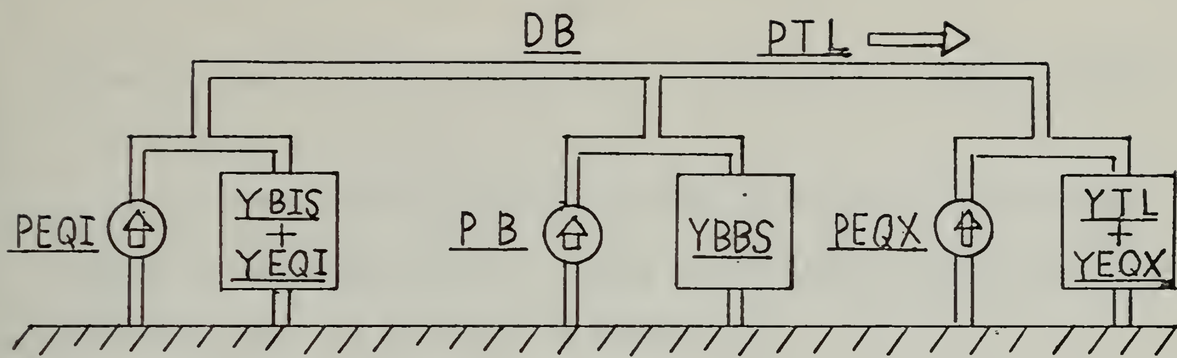
Equation (3.14a) can be considered to be two matrix Thevenin or Norton equivalent circuits, one for IS and one for XS, coupled to the circuit for BS as illustrated in Figure 3.1. If equation (3.14) is solved for $\underline{DB}$, then

$$\underline{DB} = (\underline{YEQI} + \underline{YBB} + \underline{YEQX})^{-1}(\underline{PEQI} + \underline{PB} + \underline{PEQX})$$

As shown in Appendix B, $(\underline{YEQI} + \underline{YBB} + \underline{YEQX})^{-1}$ is that part of the Whole System (WS) impedance matrix which relates $\underline{PB}$ to $\underline{DB}$. Therefore denoting $\underline{ZBB}$ for that part of the impedance matrix

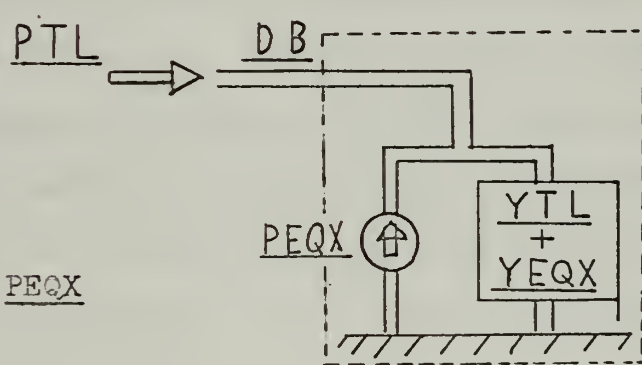
$$\underline{DB} = \underline{ZBB}(\underline{PEQI} + \underline{PB}) + \underline{ZBB}\ \underline{PEQX} \qquad (3.15)$$

In this formulation $(\underline{PEQI} + \underline{PB})$ is an input vector, $\underline{DB}$ is an output vector, and $\underline{ZBB}\ \underline{PEQX}$ is an unknown disturbance vector. Again if a



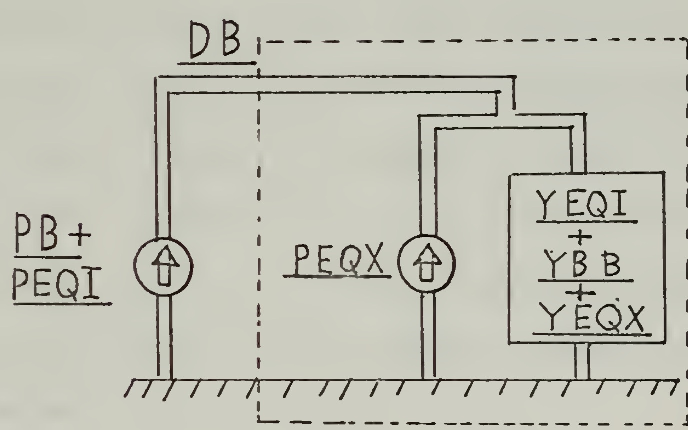
$$\underline{PEQI} + \underline{PB} + \underline{PEQX} = (\underline{YEQI} + \underline{YBIS})\underline{DB} + \underline{YBBS}\ \underline{DB} + (\underline{YEQX} + \underline{YTL})\underline{DB}$$

Figure 3.1 Equivalent Network Viewed From BS



$$\underline{PTL} = (\underline{YEQX} + \underline{YTL})\underline{DB} - \underline{PEQX}$$

Figure 3.2 Tie Line Power Flow Model



$$\underline{DB} = \underline{ZBB}(\underline{PEQI} + \underline{PB}) + \underline{ZBB}\ \underline{PEQX}$$

Figure 3.3 Boundary Bus Impedance Model

series of input, output measurements were taken, it might be possible to find $\underline{ZBB}$.

For comparison purposes and in order to be able to investigate sources of approximation error later, the analogous equations for the nonlinear model are developed in Appendix C.

3.3 Separating Own System Estimation From Equivalent System Identification

The tie line power flow model and the boundary bus impedance model are illustrated in Figures 3.2 and 3.3. As shown in Table 3.1, both approaches result in equations of the form $\underline{z} = \underline{H}\,\underline{u} + \underline{v}$ where $\underline{u}$ is the input vector, $\underline{z}$ the output vector, $\underline{v}$ a vector of unknown disturbances, and $\underline{H}$ the structure matrix to be identified. Both $\underline{z}$ and $\underline{u}$ were assumed to be known. However, since the state of OS (i.e. the bus voltages) can be estimated by knowing what power enters from XS through the tie lines, the problem of estimating the state of OS can be separated from identifying the parameters of the equivalent system. What happens to OS depends on XS, but the effect of XS is summarized by the power flowing through the tie lines. Hence existing state estimation techniques [5], [8], and [9] can be applied to obtain $\underline{z}$ and $\underline{u}$. In practice there will be some error associated with metering power and estimating voltage angles. In such a case let $\underline{v}_z$ be the error of output estimate and $\underline{v}_u$ be the error of input estimate. Then

$$(\underline{z} + \underline{v}_z) = \underline{H}(\underline{u} + \underline{v}_u) + \underline{v}$$

or

$$\underline{z} = \underline{H}\,\underline{u} + (\underline{v} - \underline{v}_z + \underline{H}\,\underline{v}_u)$$

Table 3.1

Identification Models

	General Model	Boundary Bus Impedance Model	Tie Line Power Flow Model
Output	z	DB	PTL
Input	u	(PB+PEQI)	DB
Noise	v	ZBB PEQX	-PEQX
Structure	H	ZBB	YTL+YEQX

By replacing $\underline{v} - \underline{v}_z + \underline{H}\,\underline{v}_u$ with a new unknown disturbance vector $\underline{\tilde{v}}$

$$\underline{z} = \underline{H}\,\underline{u} + \underline{\tilde{v}}$$

the same approaches can be used. The only difference is that there is generally more disturbance associated with identifying $\underline{H}$.

3.4 Use of Changes in Variables

Up to now $\underline{z}$, $\underline{u}$, and $\underline{v}$ have not been associated with time. From now on $\underline{z}(t)$, $\underline{u}(t)$, and $\underline{v}(t)$ will be used to designate a set of $\underline{z}$, $\underline{u}$, and $\underline{v}$ for a particular time t. The disturbance vector $\underline{v}(t)$ will be treated as a noise term since $\underline{u}(t)$ is analogous to an input signal and $\underline{z}(t)$ is analogous to an output signal corrupted by noise. Ordinarily when working with estimation or identification problems, it is much easier if noise terms have a zero mean. In the two approaches above neither will in general have zero temporal mean noise vectors $\underline{v}(t)$. It is expected that $\underline{v}(t)$ will vary about some value depending on the time of day, the season, etc. However, if the differences in $\underline{z}(t)$, $\underline{u}(t)$, and $\underline{v}(t)$ are taken between two different times, then it may turn out that the difference or change $\underline{v}(n) = \underline{v}(t_{n+1}) - \underline{v}(t_n)$ will have a zero mean. The general equation is then

$$\underline{z}(n) = \underline{H}\,\underline{u}(n) + \underline{v}(n) \tag{3.16}$$

where

$$\underline{z}(n) = \underline{z}(t_{n+1}) - \underline{z}(t_n)$$

$$\underline{u}(n) = \underline{u}(t_{n+1}) - \underline{u}(t_n)$$

$$\underline{v}(n) = \underline{v}(t_{n+1}) - \underline{v}(t_n)$$

It will turn out that there are other reasons for using the difference in variables rather than the actual values of the variables. The most significant is that using the actual values may result in subtracting variables whose magnitudes are very large compared to their difference.

From now on, any reference to the previously defined symbols for voltage angles or power will refer to the changes of variable between two times rather than the actual values of the variables. Hence the tie line power flow model is

$$\underline{PTL}(n) = (\underline{YTL} + \underline{YEQX})\underline{DB}(n) - \underline{PEQX}(n) \tag{3.17}$$

and the boundary bus impedance model is

$$\underline{DB}(n) = \underline{ZBB}\left[\underline{PEQI}(n) + \underline{PB}(n)\right] + \underline{ZBB}\ \underline{PEQX}(n) \tag{3.18}$$

where

$$\underline{PTL}(n) = \underline{PTL}(t_{n+1}) - \underline{PTL}(t_n)$$

$$\underline{PB}(n) = \underline{PB}(t_{n+1}) - \underline{PB}(t_n)$$

$$\underline{DB}(n) = \underline{DB}(t_{n+1}) - \underline{DB}(t_n)$$

$$\underline{PEQI}(n) = \underline{PEQI}(t_{n+1}) - \underline{PEQI}(t_n)$$

$$\underline{PEQX}(n) = \underline{PEQX}(t_{n+1}) - \underline{PEQX}(t_n)$$

for N sets of measurements $n = 1,2,\ldots,N$.

4. IDENTIFICATION OF LINEAR, STATIC SYSTEMS

4.1 Conditions on the System

Both formulations for identifying the structure of the equivalent system have resulted in the same general equation, $\underline{z}(n) = \underline{H}\ \underline{u}(n) + \underline{v}(n)$ where there are N sets of input vectors, $\underline{u}(n)$, and output vectors, $\underline{z}(n)$. $\underline{H}$ is the structure matrix and $\underline{v}(n)$ is a pseudo noise vector. The general problem can be stated:

Given: A set of input and output vectors $\underline{u}(n)$ and $\underline{z}(n)$, $n = 1,2,\ldots,N$

Find: The structure matrix $\underline{H}$ and statistical information on $\underline{v}(n)$.

Several assumptions on $\underline{u}(n)$, $\underline{v}(n)$, and $\underline{H}$ will be made and later these will be justified when the results are applied to the specific problem. First elements of $\underline{H}$ are assumed to be independent. Specifically it is assumed that $\underline{H}$ is not symmetric. This is certainly not true for the specific problem, but various justifications will be given later. Second, $\underline{v}(n)$ is assumed to be a zero temporal mean process, or $E\{\underline{v}(n)\} = \underline{0}$ where $E\{a\}$ is the expected value of a. Third, $\underline{v}(n)$ is uncorrelated in time, or $E\{\underline{v}(n)\ \underline{v}'(m)\} = \underline{0}$ for $n \neq m$ (n represents a sequence in time). Fourth, the covariance matrix for v(n) is assumed to be of the form $\frac{1}{c(n)}\underline{R}$, or $E\{\underline{v}(n)\underline{v}'(n)\} = \underline{R}(n) = \frac{1}{c(n)}\underline{R}$. c(n) can be called a confidence coefficient for the n^{th} measurement set and indicates the relative noise level for that set. Larger c(n)'s indicate less noise. Fifth, $\underline{v}(n)$ is uncorrelated with $\underline{u}(n)$, or $E\{\underline{v}(n)\underline{u}'(n)\} = \underline{0}$.

At this point there are many possible approaches to the problem. Two will be used here. One is a time tested mathematical approach which assumes a particular probability model for the noise, and the other is an engineering approach, also time tested, which uses no model

for the probability distribution of the noise. It will turn out that both result in the same solution which will have a relatively simple form. The two approaches are:

1. Assume that the noise, $\underline{v}(n)$, has a Gaussian probability distribution (with a zero mean and covariance of $\frac{1}{c(n)}\underline{R}$). Then find the maximum likelihood estimates of $\underline{H}$ and $\underline{R}$.
2. Assume no particular probability model for the noise and use the method of weighted least squares to find $\hat{\underline{H}}$. Then make a reasonable estimate for $\underline{R}$.

The fact that the two approaches result in identical solutions is a well known consequence of using quadratic criteria for optimization and estimation of linear systems. However, in addition to analytical convenience, it is also well known that in many instances the Gaussian probability distribution itself is a rather reasonable probability model.

4.2 Maximum Likelihood Identification

First the solution will be found for the maximum likelihood approach assuming Gaussian noise. There are four known items: the set of input vectors $\underline{u}(n)$, $n = 1,2,\ldots,N$, the set of output vectors, $\underline{z}(n)$, $n = 1,2,\ldots,N$, the corresponding confidence coefficients, $c(n)$, $n = 1,2,\ldots,N$, and the form of the probability distribution function for $\underline{v}(n)$, $n = 1,2,\ldots,N$, which is zero mean Gaussian, with a covariance matrix $\frac{1}{c(n)}\underline{R}$. Note that $\underline{R}$ itself is not known. Therefore if estimates of $\underline{H}$ and $\underline{R}$ can be found, the problem is solved since knowledge of $\underline{R}$ completely specifies the statistics of $\underline{v}(n)$ when $\underline{v}(n)$ is Gaussian.

The probability distribution function for the Kxl vector $\underline{v}(n)$ is

$$p[\underline{v}(n)] = \left[(2\pi)^K c^{-K}(n)|\underline{R}|\right]^{-\frac{1}{2}} e^{-\frac{1}{2}\underline{v}'(n)c(n)\underline{R}^{-1}\underline{v}(n)}$$

The likelihood function of $\underline{z}(n)$ for a given $\underline{H}$ and $\underline{R}$ when $\underline{z}(n)$ and $\underline{u}(n)$ are Kxl vectors and $\underline{H}$ and $\underline{R}$ are KxK matrices is

$$p[\underline{z}(n),\underline{u}(n):\underline{H},\underline{R}] = \left[(2\pi)\ c^{-K}(n)|\underline{R}|\right]^{-\frac{1}{2}} e^{-J[\underline{z}(n),\underline{u}(n):\underline{H},\underline{R}]}$$

where

$$J[\underline{z}(n),\underline{u}(n):\underline{H},\underline{R}] = \frac{1}{2}[\underline{z}(n) - \underline{H}\ \underline{u}(n)]'\ c(n)\ R^{-1}[\underline{z}(n) - \underline{H}\ \underline{u}(n)]$$

Since $\underline{v}(n)$ is uncorrelated in time $(E\{\underline{v}(n)\underline{v}'(m)\} = \underline{0}$ for $n \neq m)$, the joint probability distribution function for the set $\underline{V} = \{\underline{v}(1),\underline{v}(2),\ldots,\underline{v}(N)\}$ is the product of the distribution functions for $\underline{v}(1),\underline{v}(2)\ldots,\underline{v}(N)$.

$$p(\underline{V}) = p[\underline{v}(1)]\ p[\underline{v}(2)]\ \ldots\ p[\underline{v}(N)]$$

$$= \left[(2\pi)^{NK}\ |\underline{R}|^N \prod_{n=1}^{N} c^{-K}(n)\right]^{-\frac{1}{2}} \times e^{-\frac{1}{2}\sum_{n=1}^{N}\underline{v}'(n)c(n)\underline{R}^{-1}\underline{v}(n)}$$

Let $p(\underline{Z}:\underline{H},\underline{R})$ be the likelihood function of the sets $\underline{Z} = \{\underline{z}(1),\underline{z}(2),\ldots,\underline{z}(N)\}$, and $\underline{U} = \{\underline{u}(1),\underline{u}(2),\ldots,\underline{u}(N)\}$ for a given $\underline{H}$ and $\underline{R}$. Then

$$p(\underline{Z},\underline{U}:\underline{H},\underline{R}) = \left[(2\pi)^{NK}\ |\underline{R}|^N \prod_{n=1}^{N} c^{-K}(n)\right]^{-\frac{1}{2}} e^{-J(\underline{Z},\underline{U}:\underline{H},\underline{R})} \tag{4.1}$$

where

$$J(\underline{Z},\underline{U}:\underline{H},\underline{R}) = \frac{1}{2}\sum_{n=1}^{N}[\underline{z}(n) - \underline{H}\ \underline{u}(n)]'\ c(n)\ \underline{R}^{-1}[\underline{z}(n) - \underline{H}\ \underline{u}(n)]$$

Let $\hat{\underline{H}}$ and $\hat{\underline{R}}$ be the values of $\underline{H}$ and $\underline{R}$ which maximize the likelihood

function when $\underline{Z}$ and $\underline{U}$ are the actual measured sets. As developed in Appendix D these maximum likelihood estimates are

$$\hat{\underline{H}} = \left[\sum_{n=1}^{N} c(n)\underline{z}(n)\underline{u}'(n)\right]\left[\sum_{n=1}^{N} c(n)\underline{u}(n)\underline{u}'(n)\right]^{-1} \tag{4.2}$$

and

$$\hat{\underline{R}} = \frac{1}{N}\sum_{n=1}^{N} c(n)\left[\underline{z}(n) - \hat{\underline{H}}\,\underline{u}(n)\right]\left[\underline{z}(n) - \hat{\underline{H}}\,\underline{u}(n)\right]' \tag{4.3}$$

provided the vectors $\underline{u}(n)$, $n = 1,2,\ldots,N$ are such that the inverse of $\left[\sum_{n=1}^{N} c(n)\underline{u}(n)\underline{u}'(n)\right]$ exists.

Two things should be noted. First, it is not necessary to know $\hat{\underline{R}}$ in order to find $\hat{\underline{H}}$. This is significant in that it is not necessary to solve for $\hat{\underline{R}}$ and $\hat{\underline{H}}$ simultaneously which would most likely result in greatly increased computation requirements. The second is how $\hat{\underline{R}}$ varies with $\hat{\underline{H}}$. The quantity $N\left[\text{trace}(\hat{\underline{R}})\right]$ is actually $2J(\underline{Z},\underline{U}:\hat{\underline{H}},\underline{R})$ when $\underline{R}$ is set equal to the identity matrix. As seen in Appendix D, $\hat{\underline{H}}$ is the value of $\underline{H}$ such that the matrix gradient of J with respect to $\underline{H}$ is zero. Then

$$N\frac{\partial}{\partial \hat{\underline{H}}}\left[\text{trace}(\underline{R})\right] = \frac{\partial}{\partial \hat{\underline{H}}}\left[2J(\underline{Z},\underline{U}:\underline{H},\underline{R})\Big/_{\underline{R}=\underline{I}}\right] = \underline{0}$$

or in some respect $\hat{\underline{R}}$ is not sensitive to small errors in $\hat{\underline{H}}$.

4.3 Least Squares Identification

The Gaussian distribution of $\underline{v}(n)$ was assumed in order to provide a probalistic framework, however the same equations can be obtained by assuming no probability model for $\underline{v}(n)$ and instead relying on the nearly universal method of weighted least squares. The sum of weighted square errors between $\underline{z}(n)$ and $\underline{H}\ \underline{u}(n)$ is

$$S = \frac{1}{2} \sum_{n=1}^{N} \left[\underline{z}(n) - \underline{H}\ \underline{u}(n)\right]' c(n)\ \underline{T}\ \left[\underline{z}(n) - \underline{H}\ \underline{u}(n)\right]$$

where $\underline{T}$ is an unknown weighting matrix. Since S is the same as $J(\underline{Z},\underline{U}:\underline{H},\underline{R})$ when $\underline{T} = \underline{R}^{-1}$, from Appendix D it can be seen that the estimate for $\underline{H}$ did not depend on $\underline{R}$ anyway and the value of $\underline{H}$ which minimizes S for given sets $\underline{z}(1),\underline{z}(2),\ldots,\underline{z}(N)$ and $\underline{u}(1),\underline{u}(2),\ldots,\underline{u}(N)$ will be the same as the maximum likelihood estimate for $\underline{H}$. Under such circumstances, a reasonable estimate for $E\left\{c(n)\underline{v}(n)\underline{v}'(n)\right\}$ is

$$\underline{R} = E\left\{c(n)\underline{v}(n)\underline{v}'(n)\right\} \approx \frac{1}{N} \sum_{n=1}^{N} c(n)\hat{\underline{v}}(n)\hat{\underline{v}}'(n)$$

$$= \hat{\underline{R}} = \frac{1}{N} \sum_{n=1}^{N} c(n)\left[\underline{z}(n) - \hat{\underline{H}}\ \underline{u}(n)\right]\left[\underline{z}(n) - \hat{\underline{H}}\ \underline{u}(n)\right]'$$

which is the same as the maximum likelihood estimate equation (4.3).

4.4 Error Analysis

The error analysis which follows only uses the covariance of $\underline{v}(n)$. For both the approach assuming Gaussian noise and the least squares approach, the covariance of $\underline{v}(n)$ was assumed to be $E\left\{\underline{v}(n)\underline{v}'(n)\right\} = \frac{1}{c(n)}\underline{R}$. Therefore results of the error analysis are applicable to both approaches.

When the equation for $\underline{z}(n)$ is substituted into equation (4.2)

$$\hat{\underline{H}} = \left[\sum_{n=1}^{N} c(n)\underline{H}\,\underline{u}(n)\underline{u}'(n) + \sum_{n=1}^{N} c(n)\underline{v}(n)\underline{u}'(n)\right]\left[\sum_{n=1}^{N} c(n)\underline{u}(n)\underline{u}'(n)\right]^{-1}$$

$$\hat{\underline{H}} - \underline{H} = \left[\sum_{n=1}^{N} c(n)\underline{v}(n)\underline{u}'(n)\right]\left[\sum_{n=1}^{N} c(n)\underline{u}(n)\underline{u}'(n)\right]^{-1} \qquad (4.4)$$

One of the conditions on the solution was that $\underline{v}(n)$ was not correlated with $\underline{u}(n)$. It is interesting to determine the effect when that condition is not met. Let $\underline{W}$ be a weighted average of $\underline{u}(n)\underline{u}'(n)$, $n = 1,2,\ldots,N$

$$\underline{W} = \left[\sum_{n=1}^{N} c(n)\right]^{-1}\left[\sum_{n=1}^{N} c(n)\underline{u}(n)\underline{u}'(n)\right] \qquad (4.5)$$

If $\underline{v}(n)$ is correlated with $\underline{u}(n)$ so that the weighted average of $\underline{v}(n)\underline{u}'(n)$, $n = 1,2,\ldots,N$, is

$$\underline{M} = \left[\sum_{n=1}^{N} c(n)\right]^{-1}\left[\sum_{n=1}^{N} c(n)\underline{v}(n)\underline{u}'(n)\right] \approx E\left\{\underline{v}(n)\underline{u}'(n)\right\} \qquad (4.6)$$

Then $\qquad \hat{\underline{H}} - \underline{H} = \underline{M}\,\underline{W}^{-1} \qquad (4.7)$

and as N increases $\hat{\underline{H}} - \underline{H}$ will not go to zero as would be the case if $\underline{v}(n)$ were not correlated with $\underline{u}(n)$. Error is introduced in the form of a bias which is proportional to the correlation of $\underline{v}(n)$ with $\underline{u}(n)$ and inversely proportional to the "power" of $\underline{u}(n)$.

From equation (4.2) it may be noted that only the i^{th} element of

$z(n)$ affects the estimate of the i^{th} element of $\underline{H}$. Similarly from equation (4.4) only the i^{th} element of $\underline{v}(n)$ affects the i^{th} row of the error $\hat{\underline{H}} - \underline{H}$. This reflects the fact that $\underline{H}$ was assumed to be, in general, not symmetric, and that the i^{th} element of $\underline{z}(n)$ is affected only by the i^{th} row of $\underline{H}$ and the i^{th} element of $\underline{v}(n)$. Let $\underline{h}_i'$ be the i^{th} row of $\underline{H}$.

$$\underline{H} = \begin{bmatrix} \underline{h}_1' \\ \underline{h}_2' \\ \cdot \\ \cdot \\ \cdot \\ \underline{h}_K' \end{bmatrix}$$

Then the estimate of the i^{th} row of $\underline{H}$ is

$$\hat{\underline{h}}_i' = \left[\sum_{n=1}^{N} c(n) z_i(n) \underline{u}'(n)\right]\left[\sum_{n=1}^{N} c(n)\underline{u}(n)\underline{u}'(n)\right]^{-1} \tag{4.8}$$

and the error is

$$\hat{\underline{h}}_i' - \underline{h}_i' = \left[\sum_{n=1}^{N} c(n) v_i(n) \underline{u}'(n)\right]\left[\sum_{n=1}^{N} c(n)\underline{u}(n)\underline{u}'(n)\right]^{-1} \tag{4.9}$$

The error covariance between elements of the i^{th} row and j^{th} row of $\hat{\underline{H}}$ is

$$E\left\{(\underline{\hat{h}}_i - \underline{h}_i)(\underline{\hat{h}}_j - \underline{h}_j)'\right\} = \underline{P}_{ij}$$

$$= E\left\{\left[\sum_{n=1}^{N} c(n)\underline{u}(n)\underline{u}'(n)\right]^{-1}\left[\sum_{n=1}^{N} c(n)\underline{u}(n)v_i(n)\right]\right.$$

$$\left. x \left[\sum_{n=1}^{N} c(n)v_j(n)\underline{u}'(n)\right]\left[\sum_{n=1}^{N} c(n)\underline{u}(n)\underline{u}'(n)\right]^{-1}\right\}$$

$$= \left[\sum_{n=1}^{N} c(n)\underline{u}(n)\underline{u}'(n)\right]^{-1}\left[\sum_{n=1}^{N}\sum_{m=1}^{N} c(n)\underline{u}(n)\underline{u}'(m)c(m)E\left\{v_i(n)v_j(m)\right\}\right]$$

$$x \left[\sum_{n=1}^{N} c(n)\underline{u}(n)\underline{u}'(n)\right]^{-1}$$

Since $E\left\{\underline{v}(n)\underline{v}'(m)\right\} = \underline{0}$ for $n \neq m$ and $E\left\{\underline{v}(n)\underline{v}'(n)\right\} = \frac{1}{c(n)}\underline{R}$

$$\underline{P}_{ij} = E\left\{(\underline{\hat{h}}_i - \underline{h}_i)(\underline{\hat{h}}_j - \underline{h}_j)'\right\} = R_{ij}\left[\sum_{n=1}^{N} c(n)\underline{u}(n)\underline{u}'(n)\right]^{-1} \qquad (4.10)$$

Let c be the average of the c(n) 's.

$$c = \frac{1}{N}\sum_{n=1}^{N} c(n)$$

Then in terms of $\underline{W}$, the weighted average of $\underline{u}(n)\underline{u}'(n)$ defined by equation (4.5) the error covariance is

$$\underline{P}_{ij} = \frac{1}{N}\,\frac{R_{ij}}{c}\,\underline{W}^{-1} \qquad (4.11)$$

As might be expected, on the average, the error covariance of the estimate is inversely proportional to the number of measurement sets. In terms of signal to noise ratios, the quantity $c^{-1}R_{ij}$ is a measure of the power of the noise $\underline{v}(n)$, and $N\underline{W}$ is a measure of the power of the signal $\underline{u}(n)$.

5. IDENTIFICATION OF POWER SYSTEMS

The problem of identifying the equivalent admittance matrix and estimating statistics of the equivalent bus power injection vector for the external system model has been placed into a general form, and the solution to the general form has been given subject to certain conditions. The two models for identifying the equivalent system are

The Tie Line Power Flow Model

$$\underline{PTL}(n) = (\underline{YTL} + \underline{YEQX})\underline{DB}(n) - \underline{PEQX}(n)$$

The Boundary Bus Impedance Model

$$\underline{DB}(n) = \underline{ZBB}\left[\underline{PEQI}(n) + \underline{PB}(n)\right] + \underline{ZBB}\ \underline{PEQX}(n)$$

where $n = 1,2,\ldots,N$.

In both models variables such as $\underline{DB}(n)$ are the changes which occur between times t_n and t_{n+1}, $\underline{DB}(n) = \underline{DB}(t_{n+1}) - \underline{DB}(t_n)$. The general problem is

$$\underline{z}(n) = \underline{H}\ \underline{u}(n) + \underline{v}(n) \qquad n = 1,2,\ldots,N$$

$$E\left\{\underline{v}(n)\underline{v}'(n)\right\} = \frac{1}{c(n)}\ \underline{R}$$

and the solution

$$\hat{\underline{H}} = \left[\sum_{n=1}^{N} c(n)\underline{z}(n)\underline{u}'(n)\right]\left[\sum_{n=1}^{N} c(n)\underline{u}(n)\underline{u}'(n)\right]^{-1}$$

$$\hat{\underline{R}} = \frac{1}{N}\sum_{n=1}^{N} c(n)\left[\underline{z}(n) - \underline{H}\ \underline{u}(n)\right]\left[\underline{z}(n) - \underline{H}\ \underline{u}(n)\right]'$$

was based on the assumptions

1. Elements of $\underline{H}$ are independent of each other.

2. $E\{\underline{v}(n)\} = \underline{0}$

3. $E\{\underline{v}(n)\ \underline{v}'(m)\} = \underline{0}$ for $n \neq m$

4. $E\{\underline{v}(n)\ \underline{v}'(n)\} = \frac{1}{c(n)} \underline{R} \qquad n = 1,2,\ldots,N$

5. $E\{\underline{v}(n)\ \underline{u}'(n)\} = \underline{0}$

Now the effects of these assumptions on each model will be discussed.

For both models if symmetry of $\underline{H}$ were taken into account at the beginning it would be necessary to perform matrix inversions of dimension $\frac{1}{2} K(K+1)$ where K is the dimension of $\underline{H}$. In developing the error covariance equation for $\hat{\underline{H}}$ it was noted that with $\underline{H}$ not symmetric, each row of $\underline{H}$ is actually identified independently of the other rows of $\underline{H}$. With symmetry this is no longer true and the result is that all independent elements of $\underline{H}$ (either upper or lower diagonal must be identified together. The size of the matrix inversion necessary can be significant as the dimension of $\underline{H}$ increases. For example, if the dimension of $\underline{H}$ is 10, the dimension of the matrix inversion is 10 when $\underline{H}$ is not assumed to be symmetric because as was shown earlier, each row of $\underline{H}$ is identified independently of the other rows and only one matrix is inverted. However, an inversion of dimension 55 is required when symmetry is accounted for since the 55 independent elements must be identified together. One way to handle symmetry at the end is to treat the ij^{th} element and the ji^{th} element of $\hat{\underline{H}}$ as two estimates of the ij^{th} element of $\underline{H}$ (for $i \neq j$). Then using the corresponding error variances, the weighted estimates can be combined. Let a be the ij^{th} element of $\hat{\underline{H}}$ and r its error variance, and b the ji^{th} element of $\hat{H}$

and q its error variance, then the combined estimate c is

$$c = \frac{\frac{a}{r} + \frac{b}{q}}{\frac{1}{r} + \frac{1}{q}}$$

Symmetry of $\underline{\underline{ZBB}}$ and $\underline{\underline{YEQX}}$ is merely the result of reciprocity since they represent passive networks (the sources, $\underline{PEQI}$, $\underline{PB}$, and $\underline{PEQX}$ have been moved outside). However, in the case of $\underline{\underline{ZBB}}$ what is ultimately important is the product $\underline{\underline{ZBB}}(\underline{PEQI} + \underline{PB})$. By the principle of reciprocity an input at terminal 1 causes an output at terminal 2 equal to the output at terminal 1 caused by the same input at terminal 2. But because of different demands throughout the network, the input at terminal 2 may never be as large as the input at terminal 1.

Had the actual power injection and voltage angles been used, $\underline{v}(t_n)$ (i.e. $\underline{PEQX}(t_n)$ or $\underline{\underline{ZBB}}\ \underline{PEQX}(t_n)$) would most likely have a non zero mean value. However, because changes in values were used, it may be expected that $\underline{v}(n)$ will have a zero mean value provided all the measurements are not taken when the entire system is moving in the same direction. For the case when the entire system is moving in the same direction, it may turn out that $E\left\{\underline{v}(n)\underline{v}'(n-k)\right\} = \underline{0}$ for k greater than 1 or 2. That is, $\underline{v}(n)$ is only correlated with only the last one or two measurement sets. This could be accounted for by a modification to the approach, but it is not anticipated that time correlation of $\underline{v}(n)$ will be a major source of error. Also, when the entire system is moving in one general direction, depending on the sampling interval, there will still probably be some independent movement of certain buses in the opposite direction due to base loaded generators, load fluctuations, etc.

The form of $\underline{R}(n) = E\{\underline{v}(n)\underline{v}'(n)\} = \frac{1}{c(n)}\underline{R}$ was based on the assumption that the covariance of the noise is dependent only on the time interval between measurement sets $\underline{z}(t_n)$, $\underline{u}(t_n)$ and $\underline{z}(t_{n+1})$, $\underline{u}(t_{n+1})$. It is then reasonable to assume that the covariance of $\underline{v}(n) = \underline{v}(t_{n+1})-\underline{v}(t_n)$ is of the form $\underline{R}(n) = (t_{n+1} - t_n)\underline{R}$. There may be error in the measurements $\underline{z}(n)$ and $\underline{u}(n)$ but it was shown how such error could be lumped with $\underline{v}(n)$. If $\underline{v}_z(n)$ and $\underline{v}_u(n)$ are the errors of $\underline{z}(n)$ and $\underline{u}(n)$ respectively, then $\underline{z}(n) = \underline{H}\ \underline{u}(n) + \tilde{\underline{v}}(n)$

where $\tilde{\underline{v}}(n) = -\underline{v}_z(n) + \underline{H}\ \underline{v}_u(n) + \underline{v}(n)$

Since $\underline{v}(n)$ is independent of $\underline{v}_z(n)$ and $\underline{v}_u(n)$, $E\{\tilde{\underline{v}}(n)\tilde{\underline{v}}'(n)\} = \underline{Q} + \frac{1}{c(n)}\underline{R}$ where $\underline{Q} = E\left\{[\underline{H}\ \underline{v}_u(n) - \underline{v}_z(n)]\ [\underline{H}\ \underline{v}_u(n) - \underline{v}_z(n)]'\right\}$ and should be relatively constant since it depends on measurement error. The measurement error should be in the vicinity of one to ten percent while $\underline{v}(n)$ can be expected to be in the vicinity of 100% of $\underline{H}\ \underline{u}(n)$ so the effect of $\underline{Q}$ on the estimate of $\underline{R}$ should be very minor.

The last assumption, that $\underline{v}(n)$ is uncorrelated with $\underline{u}(n)$ can be a source of considerable error when this condition is not met. The error between $\hat{\underline{H}}$ and $\underline{H}$ was shown to be

$$\hat{\underline{H}} - \underline{H} = \left[\sum_{n=1}^{N} c(n)\underline{v}(n)\underline{u}'(n)\right]\left[\sum_{n=1}^{N} c(n)\underline{u}(n)\underline{u}'(n)\right]^{-1}$$

For the boundary bus impedance model

$$\underline{DB}(n) = \underline{ZBB}[\underline{PEQI}(n) + \underline{PB}(n)] + \underline{ZBB}\ \underline{PEQX}(n)$$

$$\underline{v}(n)\underline{u}'(n) = \underline{ZBB}\ \underline{PEQX}(n)[\underline{PEQI}(n) + \underline{PB}(n)]'$$

$$= \underline{ZBB}\ \underline{AQX}[\underline{PX}(n)\underline{PI}'(n)\underline{AQI}' + \underline{PX}(n)\underline{PB}'(n)]$$

while for the tie line power flow model

$$\underline{PTL}(n) = (\underline{YTL} + \underline{YEQX})\underline{DB}(n) - \underline{PEQX}(n)$$

$$\underline{v}(n)\underline{u}'(n) = -\underline{PEQX}(n)\underline{DB}'(n)$$

$$= -\underline{AQX}\ \underline{PX}(n)\left[\underline{AQI}\ \underline{PI}(n) + \underline{PB}(n) + \underline{AQX}\ \underline{PX}(n)\right]'\ \underline{ZBB}$$

$$= -\underline{AQX}\left[\underline{PX}(n)\underline{PI}'(n)\underline{AQI}' + \underline{PX}(n)\underline{PB}'(n)\right]\ \underline{ZBB}$$

$$- \underline{AQX}\ \underline{PX}(n)\underline{PX}'(n)\underline{AQX}'\underline{ZBB}$$

where $\underline{AQX}$ and $\underline{AQI}$ were defined in equations (3.7) and (3.13). Both models have the term $\underline{AQX}\left[\underline{PX}(n)\underline{PI}'(n)\underline{AQI} + \underline{PX}(n)\underline{PB}'(n)\right]$ in common. However, the tie line power flow model has the additional term $\underline{AQX}\ \underline{PX}(n)\underline{PX}'(n)\underline{AQX}' = \underline{PEQX}(n)\underline{PEQX}'(n)$. Ordinarily it would be expected that a change in power injection at one particular bus will be independent of the changes in power injections at all other buses so that $E\left\{\underline{PX}(n)\underline{PI}'(n)\right\} = \underline{0}$ and $E\left\{\underline{PX}(n)\underline{PB}'(n)\right\} = \underline{0}$. The exceptions might occur when the entire system is moving in the same direction as discussed earlier. However in the case of the tie line power flow model, $E\left\{\underline{PEQX}(n)\underline{PEQX}'(n)\right\} = \underline{R}$ is not zero unless the external system bus injections do not change. This causes a bias in the estimate of $\underline{H}$ which depends on the magnitude of $\underline{PEQX}(n)$ in relation to $\underline{PEQI}(n) + \underline{PB}(n)$, but in general it can be expected that the two will be of comparable magnitude so that the bias introduced is in the vicinity of 100%. Using $\underline{W}$ and $\underline{M}$ as defined by equations (4.5) and (4.6)

$$\underline{M} = \left[\sum_{n=1}^{N} c(n)\right]^{-1} \left[\sum_{n=1}^{N} c(n)\underline{v}(n)\underline{u}'(n)\right]$$

$$\approx -\left[\sum_{n=1}^{N} c(n)\right]^{-1} \left[\sum_{n=1}^{N} c(n)\underline{PEQX}(n)\underline{PEQX}'(n)\right] \underline{ZBB}$$

$$\approx -\underline{R}\ \underline{ZBB}$$

the bias in the estimate for the tie line power flow model is

$$\underline{\hat{H}} - \underline{H} = \underline{M}\,\underline{W}^{-1} \approx -\underline{R}\,\underline{\overline{ZBB}}\,\underline{W}^{-1}$$

so that the estimation equations for $\underline{\hat{H}}$ and $\underline{\hat{R}}$ are really coupled for this model.

The boundary bus impedance model and the tie line power flow model are two specific approaches to the problem of identifying the equivalent external system model. In the former the inputs are Own System bus power injections and the outputs are boundary bus voltage angles. In the latter the inputs are boundary bus voltage angles while outputs are tie line power flows (which depends on voltage angles). Because it is power which is bought and sold, while voltage angles are determined by the line admittances and the distribution of bus power injections, it can be seen that bus power injections are the legitimate independent inputs and not voltage angles. This is the cause of the bias in the estimate for the tie line power flow model. Both models need $\underline{DB}(n)$ but the tie line power flow model uses the tie line power flow vector, $\underline{PTL}(n)$, which in general will be more accurate a measurement than the estimate for the equivalent injections from IS, $\underline{PEQI}(n)$. It is possible to measure $\underline{PTL}(n)$ directly while the actual value of $\underline{PEQI}(n)$ is dependent on voltage magnitudes and transmission line resistance as shown by the nonlinear analysis in Appendix C. However the solutions for $\underline{\hat{H}}$ and $\underline{\hat{R}}$ are more readily found for the boundary bus impedance model.

Obtaining the estimate of $\underline{YEQX}$ from the estimate of $\underline{ZBB}$ is straightforward. However, if the estimate of $\underline{YEQX}$ is to ultimately be used in a linear load flow model for OS, then as shown in Appendix B, the estimate for $\underline{ZBB}$ can be used directly. If $\underline{ZOS}$ is the bus impedance

matrix for CS,

$$\begin{bmatrix} \underline{DI} \\ \underline{DB} \end{bmatrix} = \begin{bmatrix} \underline{ZCS} \\ \ \end{bmatrix} \begin{bmatrix} \underline{PI} \\ \underline{PB} + \underline{PEQX} \end{bmatrix}$$

where

$$\underline{ZCS} = \begin{bmatrix} \underline{YII}^{-1} & \underline{0} \\ \underline{0} & \underline{0} \end{bmatrix} + \begin{bmatrix} \underline{AQI}' \\ \underline{I} \end{bmatrix} \underline{ZBB} \begin{bmatrix} \underline{AQI} & \underline{I} \end{bmatrix}$$

6. SIMULATION RESULTS

In order to test the method proposed, a computer simulation program was developed to generate data, identify the equivalent model, and check how well the identified model could predict line flow changes in OS. Figure 6.1 gives a broad overview of how the simulation was conducted. First, using transmission line data, matrices to be used in the simulation were computed. These include $\underline{ZWS}$ (impedance matrix for Whole System), $\underline{AQI}$, $\underline{YII}^{-1}$, ($\underline{YEQI}$ + $\underline{YBOS}$), ($\underline{YEQX}$ + $\underline{YTL}$), $\underline{AQX}$, and $\underline{YTL}$. For convenience, the confidence coefficients, c(1), c(2),..., c(N), were all unity. Matrices $\underline{CHI}$ and $\underline{SGINV}$ accumulated a running sum of $\underline{z}(n)\underline{u}'(n)$ and $\underline{u}(n)\underline{u}'(n)$ respectively. In order to simulate the variations in bus power injections throughout the network, $\underline{P}(n)$ (vector of bus power injection changes for WS) was generated by random numbers having an RMS (root mean square) value which was a fraction of the nominal operating value of bus power injections $\underline{P}_{nom}$. One fraction, PCTOS, was used for all buses in OS and another, PCTXS, was used for all buses in XS. (i.e. $E\left\{PX_i^2(n)\right\} = \left[PCTXS \times PX_{nom_i}\right]^2$) This maintained a relative scale for changes while at the same time it simulated independent changes in power injections. As shown in Section 3, when the linear model is used, the bus voltage angle vector for OS can be found using either

$$\begin{bmatrix} \underline{DI}(n) \\ \underline{DB}(n) \\ \underline{DX}(n) \end{bmatrix} = \underline{ZWS} \begin{bmatrix} \underline{PI}(n) \\ \underline{PB}(n) \\ \underline{PX}(n) \end{bmatrix} \tag{6.1}$$

or equivalently

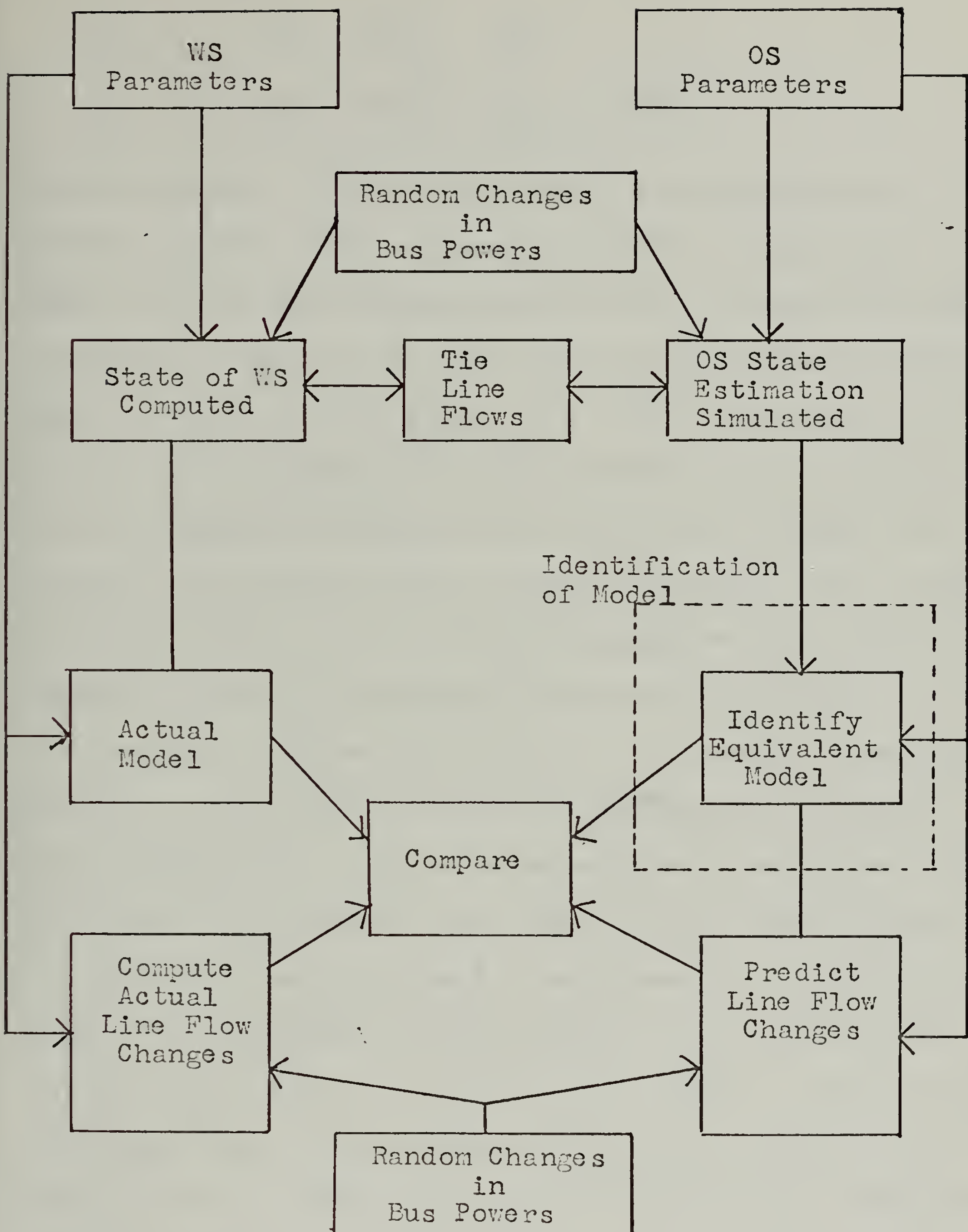


Figure 6.1 Basic Flow Diagram For Simulation

$$\begin{bmatrix} \underline{DI}(n) \\ \\ \underline{DB}(n) \end{bmatrix} = \begin{bmatrix} \underline{YII} & \underline{YIB} \\ \\ \underline{YIB}' & \underline{YBOS} \end{bmatrix}^{-1} \begin{bmatrix} \underline{PI}(n) \\ \\ \underline{PB}(n) - \underline{PTL}(n) \end{bmatrix} \tag{6.2}$$

Ordinarily equation (6.2) or its nonlinear analog would be used to estimate the state of OS using real measurements of $\underline{PI}(n)$, $\underline{PB}(n)$, and $\underline{PTL}(n)$ (and any other redundant measurements). However, for purposes of simulation equation (6.1) would have to be solved first to find $\underline{DX}(n)$ in order to generate $\underline{PTL}(n)$. Since either method results in the same values of $\underline{DB}(n)$ (in the linear model), equation (6.1) was used to simulate the output of a state estimation for $\underline{DB}(n)$. The simulated state estimation for OS results in errorless data. However as discussed earlier the only effect of measurement noise and state estimation errors is to modify the disturbance vector $\underline{v}(n)$.

Everything described up to now was for the purpose of generating data for the identification routine. Using generated data, $\underline{PI}(n)$, $\underline{PB}(n)$, and $\underline{DB}(n)$, the input and output measurement vectors were formed, $\underline{u}(n) = \underline{AQI}\ \underline{PI}(n) + \underline{PB}(n)$ and $\underline{z}(n) = \underline{DB}(n)$. These values were also stored for later use in finding $\hat{\underline{R}}$. Running sums of $\underline{z}(n)\underline{u}'(n)$ and $\underline{u}(n)\underline{u}'(n)$ were maintained by $\underline{CHI}$ and $\underline{SGINV}$ so that at any time the current estimate $\hat{\underline{H}}$ could be found by $\underline{H} = \underline{CHI}\ \underline{SGINV}^{-1}$. Using $\hat{\underline{H}}$ and the stored measurement sets, an estimate $\hat{\underline{R}}$ was found. Parallel to all of this, the actual disturbance vector, $\underline{v}(n)$, found by $\underline{v}(n) = \underline{ZBB}\ \underline{PEQX}(n) = \underline{ZBX}\ \underline{PX}(n)$, was used to maintain a running sum of $\underline{v}(n)\underline{v}'(n)$ in $\underline{R}$. The estimated values $\hat{\underline{H}}$ and $\hat{\underline{R}}$ could then be compared to the actual values $\underline{H} = \underline{ZBB}$ and $\underline{R}$. In order to compare how well the identified model could predict changes in line power flows, the estimate for $\underline{ZOS}$,

the impedance matrix relating OS bus power changes to OS bus voltage angle changes was found by

$$\underline{\widehat{ZOS}} = \begin{bmatrix} \underline{YII}^{-1} & \underline{0} \\ \underline{0} & \underline{0} \end{bmatrix} + \begin{bmatrix} \underline{AQI}' \\ \underline{I} \end{bmatrix} \underline{\widehat{ZBB}} \begin{bmatrix} \underline{AQI} & \underline{I} \end{bmatrix} \tag{6.3}$$

Again random variations of bus power injections were generated using a nominal base value and estimated bus voltage angle changes found by

$$\begin{bmatrix} \underline{\widehat{DI}}(n) \\ \underline{\widehat{DB}}(n) \end{bmatrix} = \underline{\widehat{ZOS}} \begin{bmatrix} \underline{PI}(n) \\ \underline{PB}(n) \end{bmatrix} \tag{6.4}$$

were used to estimate line flow changes. The actual bus voltage angle changes using the same bus power changes were found from $\underline{D}(n) = \underline{ZWS}\ \underline{P}(n)$ and were used to find actual line flow changes which could be compared to the values found using the identified model.

Throughout the program, in order to present data in a condensed form, for many variables ($\underline{z}(n)$, $\underline{u}(n)$, $\underline{v}(n)$, actual line flows, errors in predicted line flows, and generated bus power changes) only the RMS value, the mean, the minimum, and the maximum values were maintained. It should also be pointed out that the basic approach is simple. Only a small portion of the program was involved in the actual identification of $\hat{\underline{H}}$ and $\hat{\underline{R}}$. The rest is support for generating data and evaluating results.

Using equation (4.11), the estimated error covariance matrix, $\hat{\underline{P}}_{ii}$, of row i of $\underline{H}$ is for c(n) = 1.0

$$\hat{\underline{P}}_{ii} = \frac{1}{N} \hat{R}_{ii} \underline{W}^{-1}$$

where

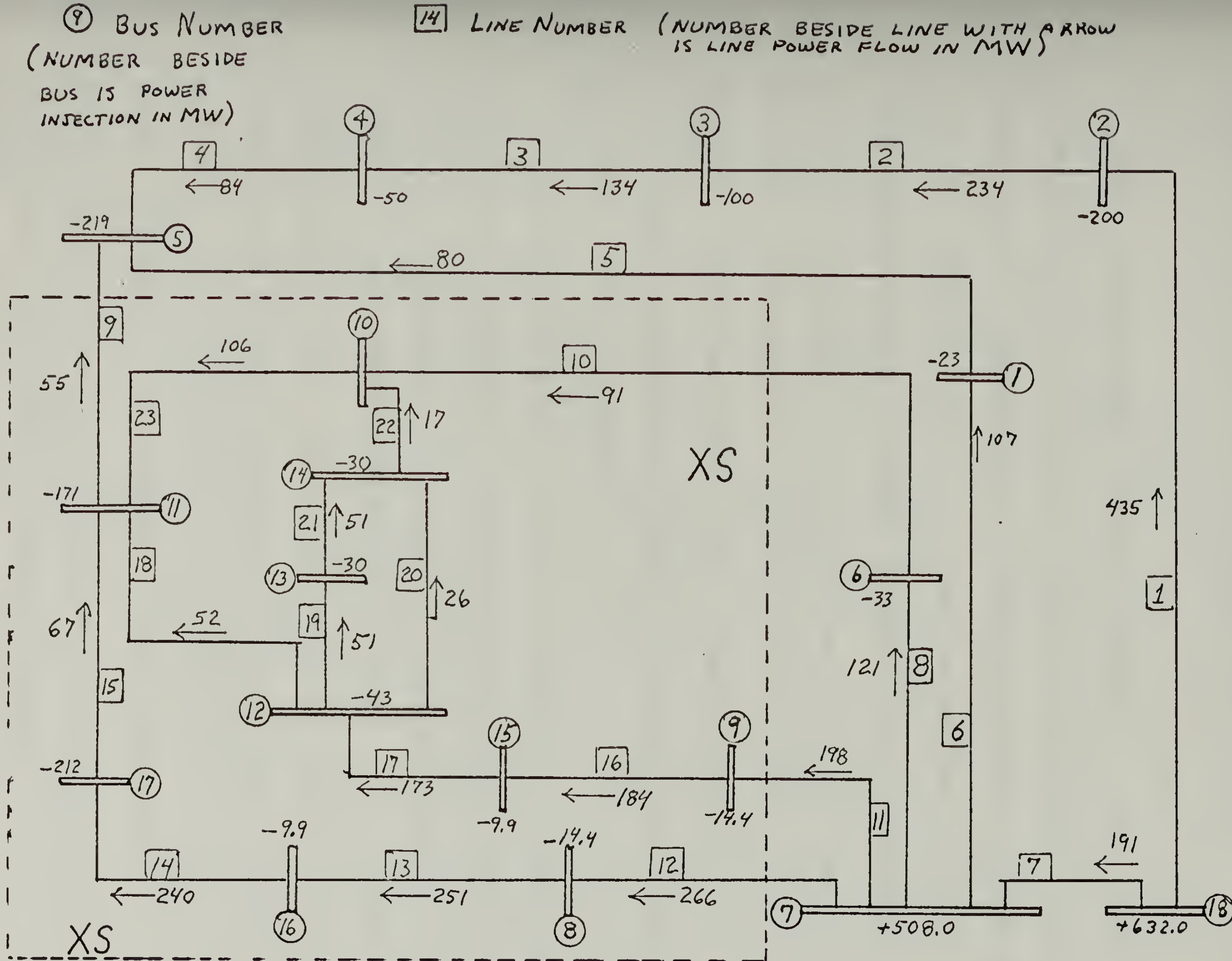
$$\underline{W}^{-1} = \left[\frac{1}{N}\sum_{n=1}^{N} \underline{u}(n)\underline{u}'(n)\right]^{-1} = N \times \underline{SGINV}^{-1} = N \times \underline{SIGMA}$$

Hence the estimated standard deviation of element ij of $\underline{H}$ is $(\hat{R}_{ii} \times SIGMA_{jj})^{\frac{1}{2}}$. Since $\hat{R}_{ii}$ should be somewhat proportional to the square of PCTXS while $SIGMA_{jj}$ should be inversely proportional to the square of PCTCS, in the case of the linear model with perfect measurements of $\underline{z}(n)$ and $\underline{u}(n)$, identification error is dependent on the ratio of PCTXS to PCTCS (percentage variation of XS bus powers to percentage variation of CS bus powers).

One system presented in a paper [2] was used here to test the method. The system shown in Figure (6.2) consists of 18 buses. For purposes of simulation Whole System consists of these 18 buses of which buses 1,2,3,4, and 18 belong to IS, 5 through 7 belong to BS, and 8 through 17 belong to XS. Bus 18 is the reference bus. Other partitionings could be used but this one does result in matrices ($\underline{YEQX}$ + $\underline{YTL}$) and ($\underline{YEQI}$ + $\underline{YBCS}$) which are comparable in size. This avoids a case in which coupling of BS with IS is very much stronger than BS with XS. For this system ($\underline{YEQI}$ + $\underline{YBCS}$) and ($\underline{YEQX}$ + $\underline{YTL}$) are

$$\underline{YEQI} + \underline{YBCS} = \begin{bmatrix} 13.63 & 0.0 & -4.07 \\ 0.0 & 9.43 & -9.43 \\ -4.07 & -9.43 & 85.9 \end{bmatrix}$$

$$\underline{YEQX} + \underline{YTL} = \begin{bmatrix} 11.5 & -6.14 & -5.36 \\ -6.14 & 14.3 & -8.12 \\ -5.36 & -8.12 & 13.5 \end{bmatrix}$$



1	52.6
2	60.5
3	28.5
4	12.22
5	5.77
6	13.8
7	72.4
8	9.43
9	16.29
10	28.4
11	34.7
12	34.7
13	62.9
14	65.0
15	25.8
16	63.3
17	27.1
18	18.12
19	188.8
20	18.05
21	17.35
22	111.2
23	86.3

Figure 6.2 Network Used For Simulation

Both the effects of increasing disturbances and increasing measurements are simulated here. In Figure 6.3 and Table 6.1 are the results of increasing disturbances ($\underline{PX}(n)$). The RMS of bus power changes for OS in each case was 10% of nominal while RMS values of $\underline{PX}(n)$ were 0%, 1.25%, 2.5%, 5.0%, 10%, and 20% of $\underline{PX}_{nom}$. Another way to say this is that "signal to noise ratios" were ∞:1, 8:1, 4:1, 2:1, 1:1, and 1:2 respectively. The same sequence of random numbers were generated each time. Included in Table 6.1 are actual values of $\underline{ZBB}$, actual errors in identifying it, and the estimated standard deviations using $\hat{\underline{R}}$ in equation (4.11). Also included are the values of $(\widehat{\underline{YEQX} + \underline{YTL}})$ estimated from $\widehat{\underline{ZBB}}$, and the errors of predicting changes in line flows using $\widehat{\underline{ZBB}}$. In a similar format, the results of increasing the number of measurements are summarized in Figure 6.4 and Table 6.2. Changes in both OS and XS bus power injections had RMS values of 10% of the nominal operating point while $\hat{\underline{H}}$ was identified using 16, 32, 64, 128, 256, and 512 measurements.

The results show that not only is identification error commensurate with the noise and number of measurements, but also estimated standard deviation gives an accurate measure of the identification error. Because the same random numbers were generated for each value of PCTXS in the first case, the actual errors shown in Figure 6.3 are nearly exactly linear since as was shown earlier, for equal numbers of measurements identification error depends on the ratio of PCTXS to PCTOS. For the case of increasing measurements in Figure 6.4, ZBB_{11} does not have a steadily decreasing error as with ZBB_{22}, but it is still in the range of the estimated standard deviation.

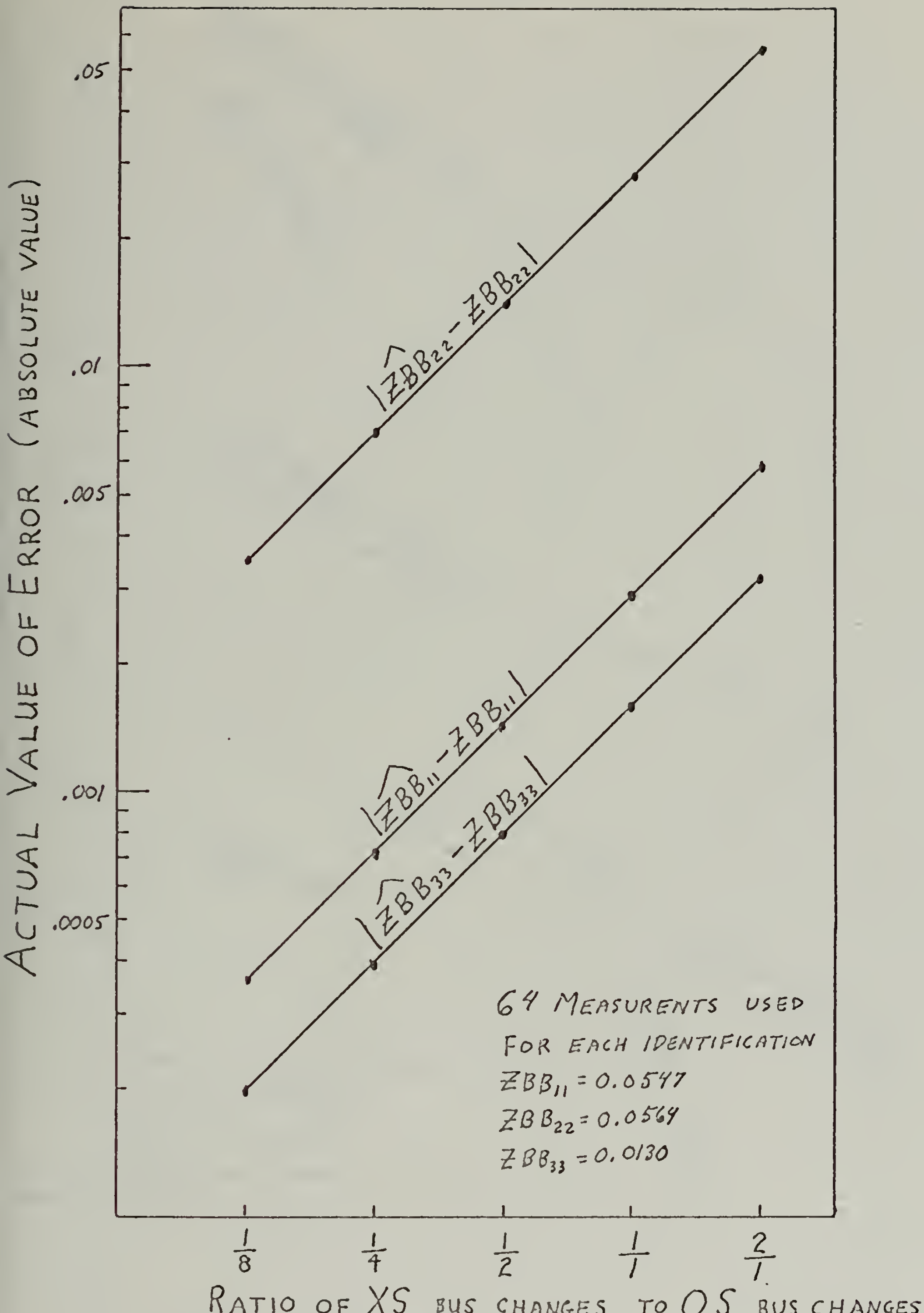


Figure 6.3 Identification Error For Increasing Disturbances

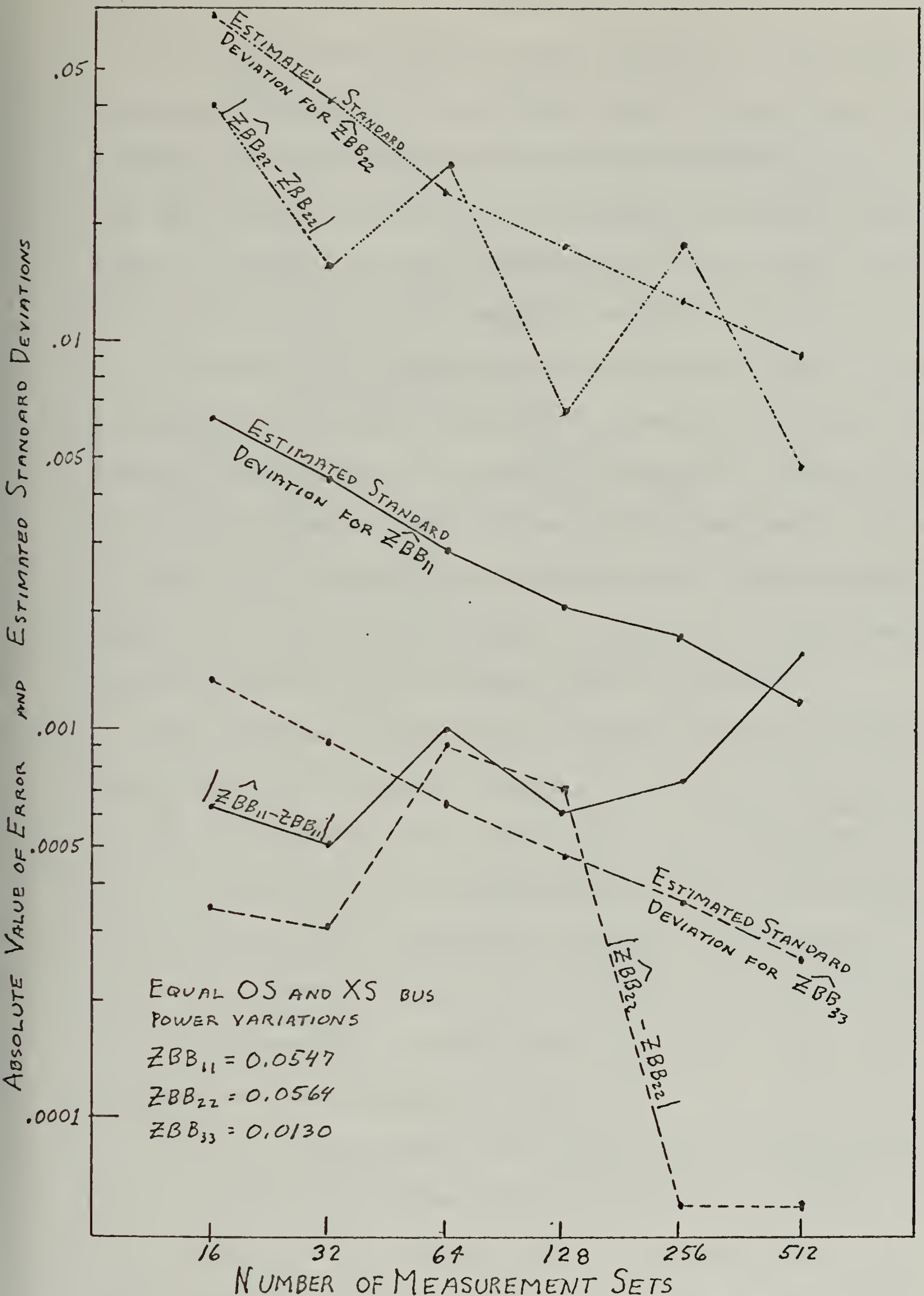


Figure 6.4 Identification Error For Increasing Measurements

To contrast identification models, in Table 6.3 the results of identifying $\widehat{\underline{YEQX}} + \underline{YTL}$ by the tie line power flow model along with the estimated standard deviations are compared with $\widehat{\underline{YEQX}} + \underline{YTL}$ as found from $\widehat{\underline{ZBB}}$ which was identified by the boundary bus impedance model using the same data. Besides identifying the wrong values, the tie line power flow model gives inaccurate estimates of error.

In Figure 6.5 is another network presented in a paper by Stagg [9]. OS consists of the higher voltage network and XS is the lower voltage network. The "tie lines" are really transformers. Using the boundary bus impedance model an equivalent network is identified to replace the 21 buses of XS, so that OS can be analyzed with 9 buses instead of the original 30. The results in Table 6.4 again show close correspondence between actual error and estimated standard deviation.

The simulation carried out was certainly not fully realistic. However the realistic aspects include

1. Values of bus power changes were found first, then a state estimation for OS was simulated and appropriate values were passed to the identification portion of the program.
2. The effects of increasing measurements were studied under the assumption that XS would vary as much as OS.

The simulation was unrealistic in that

1. The nonlinear model was not used to generate data with the result that a linear identification model was used to find parameters to a linear system. To generate data from a nonlinear model properly would require a complete load flow for each measurement set. (It would be simple to vary voltage magnitude and voltage angle and then find bus power, but bus

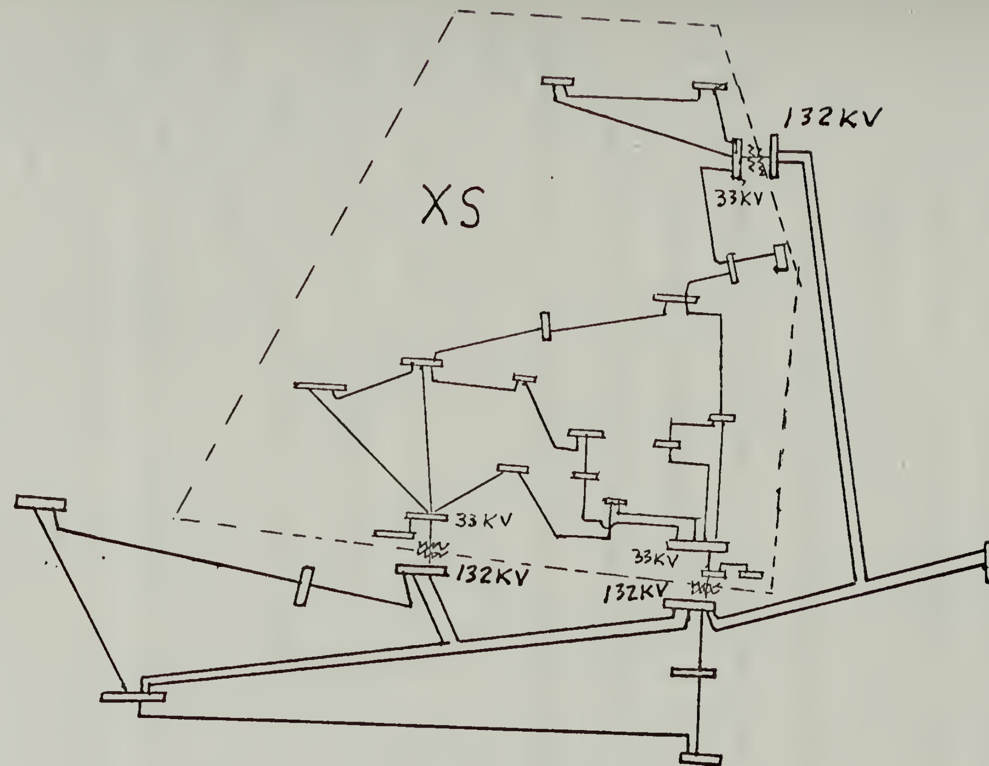


Figure 6.5 Network Used For Identification of a Lower Voltage Subsystem

power must be the specified variable as it is the independent input, not voltage angle.)

2. Bus power injections were independently generated whereas realistically they will probably be correlated in space and time. There were no conditions on the solution for correlation in space. In fact $\underline{PEQX}(n)$ itself has a covariance matrix $E\left\{\underline{PEQX}(n)\underline{PEQX}'(n)\right\}$ which is in general not diagonal. However, the solution did assume no correlation in time. As discussed earlier in Section 5, this effect can be accounted for by modifying the approach.

Based on the results of these and other simulations certain conclusions can be made about the boundary bus impedance model. First, with enough measurements it is possible to identify the equivalent model under conditions of the simulation (independent movement of the bus injections). The number of measurements may seem rather large but of course the noise is also rather large. The results show that as expected either a decrease in the disturbances by a half or using four times as many measurements approximately halves the error. Second and possibly even more important, the error predicted by using the estimated $\hat{\underline{R}}$ in equation (4.11) agrees very well with the actual answer. This means that an accurate estimate can be predicted from only the measurements used to identify $\hat{\underline{H}}$ so that the limits to which $\hat{\underline{H}}$ can be trusted are also known. This is of course not the case with the tie line power flow model.

A very important question which may arise is how much data would be required to identify the system. The simulation was carried out under idealized circumstances and as was shown earlier, for the

simulation the identification error for equal numbers of measurements depends only on the ratio of XS bus power changes to CS bus power changes. In actuality however there are upper and lower limits. If data were used from samples taken too close together in time so that changes were small, the effect of measurement noise and state estimation error could become very significant. On the other hand, the system can vary over only so much of a range, so that if the time between measurements were too large, then because of cyclical patterns of power requirements, there might not be much new information after the first few measurements. To determine the amount of data required and the best interval between samples would require a study using actual operating data.

Identified $\underline{\hat{H}} = \widehat{\underline{ZBB}}$

PCTXS	Element	Row 1 ZBB_{11}	ZBB_{12}	ZBB_{13}	Row 2 ZBB_{21}	ZBB_{22}	ZBB_{23}	Row 3 ZBB_{31}	ZBB_{32}	ZBB_{33}
	Actual Value	.05472	.02075	.00885	.02075	.05643	.01193	.00885	.01193	.01301
0.0%	A.E.	.0	.0	.0	.0	.0	.0	.0	.0	.0
	E.S.D.	.0	.0	.0	.0	.0	.0	.0	.0	.0
1.25%	A.E.	.00036	-.00302	-.00036	.00040	-.00352	-.00043	.00020	-.00148	-.00020
	E.S.D.	.00036	.00260	.00015	.00043	.00309	.00018	.00018	.00126	.00007
2.5%	A.E.	.00073	-.00604	-.00072	.00080	-.00703	-.00086	.00040	-.00295	-.00040
	E.S.D.	.00073	.00520	.00030	.00087	.00617	.00035	.00035	.00251	.00014
5.0%	A.E.	.00146	-.01207	-.00144	.00161	-.01406	-.00172	.00080	-.00590	-.00080
	E.S.D.	.00146	.01040	.00059	.00174	.01235	.00071	.00070	.00502	.00029
10.0%	A.E.	.00292	-.02414	-.00289	.00322	-.02812	-.00345	.00159	-.01181	-.00159
	E.S.D.	.00292	.02079	.00119	.00347	.02470	.00141	.00141	.01004	.00057
20.0%	A.E.	.00584	-.04828	-.00577	.00643	-.05625	-.00690	.00318	-.02362	-.00318
	E.S.D.	.00584	.04158	.00238	.00694	.04940	.00282	.00282	.02008	.00115

A.E. -- Actual Error of Identified Values
E.S.D. -- Estimated Standard Deviation using $\underline{\hat{R}}$
PCTXS x $\underline{PX}_{nom}$ is the RMS value of XS bus power changes

RMS variation of bus power changes in OS 10% of nominal values
64 Measurement sets used each time

Table 6.1(a) Identification Error for Increasing Disturbances ($\underline{ZBB}$)

$\widehat{\underline{YEQX}} + \underline{YTL}$

PCTXS	Element	Row 1: 1,1	1,-	1,3	Row 2: 2,1	2,2	2,3	Row 3: 3,1	3,2	3,3
	Actual Value	11.50	-6.14	-5.36	-6.14	14.26	-8.12	-5.36	-8.12	13.47
0.0%	A.E.	.00	.00	.00	.00	.00	.00	.00	.00	.00
1.25%	A.E.	-.32	.73	-.17	-.38	.90	-.20	-.66	1.30	-.04
2.5%	A.E.	-.67	1.53	-.36	-.80	1.89	-.32	-1.37	2.71	-.11
5.0%	A.E.	-1.47	3.36	-.78	-1.75	4.16	-.92	-2.99	6.00	-.33
10.0%	A.E.	-3.67	8.40	-1.90	-4.46	10.47	-2.17	-7.34	14.12	-1.24
20.0%	A.E.	-14.89	34.16	-7.18	-18.53	42.99	-8.83	-28.78	62.52	-8.70

$(\widehat{\underline{YEQX}} + \underline{YTL})$ found from $\widehat{\underline{ZBB}}^{-1} - (\underline{YEQI} + \underline{YBOS})$

A.E. -- Actual Error of Identified Values

PCTXS x $\underline{PX}_{nom}$ is the RMS value of XS bus power changes

RMS variation of bus power changes in OS 10% of nominal values

64 Measurement sets used each time

Table 6.1(b) Identification Error For Increasing Disturbances ($\underline{YEQX}$ + $\underline{YTL}$)

Errors in Predicting Line Power Flow Changes

PCTXS	LINE	1	2	3	4	5	6	7	8
	Nominal Line Power Flow	433.6	233.6	133.6	83.6	79.3	102.6	177.4	124.7
	RMS Actual Change	12.703	7.126	4.956	5.003	2.090	2.378	30.570	1.324
0%	RMS Error	.000	.000	.000	.000	.000	.000	.000	.000
1.25%	RMS Error	.080	.080	.080	.080	.023	.023	.472	.075
2.5%	RMS Error	.161	.161	.161	.161	.047	.047	.953	.150
5.0%	RMS Error	.321	.321	.321	.321	.094	.094	1.887	.300
10.0%	RMS Error	.643	.643	.643	.643	.187	.187	3.774	.600
20.0%	RMS Error	1.286	1.286	1.286	1.286	.374	.374	7.549	1.199

RMS Actual Change is the root mean square value of 20 changes in bus power pattern in OS.

RMS Error is the root mean square error in predicting line flow changes using the identified models corresponding to Table 6.1(a).

Changes in OS bus powers were generated by uniformly distributed random numbers such that OS powers varied up to ±10% of nominal values.

Table 6.1(c) Identification Error for Increasing Disturbances (Line Flows)

Identified $\hat{\underline{H}} = \underline{\widehat{ZBB}}$

Number of Measurement Sets	Element	Row 1			Row 2			Row 3		
		ZBB_{11}	ZBB_{12}	ZBB_{13}	ZBB_{21}	ZBB_{22}	ZBB_{23}	ZBB_{31}	ZBB_{32}	ZBB_{33}
	Actual Value	.05472	.02075	.00885	.02075	.05643	.01193	.00885	.01193	.01301
16	A.E.	.00064	-.03571	-.00021	.00062	-.04029	-.00021	.00018	-.02337	-.00035
	E.S.D.	.00644	.05711	.00281	.00767	.06803	.00335	.00306	.02718	.00134
32	A.E.	.00051	-.01291	-.00035	.00026	-.01569	-.00047	.00028	-.00999	-.00031
	E.S.D.	.00441	.03480	.00193	.00527	.04157	.00231	.00213	.01678	.00093
64	A.E.	-.00099	-.02263	-.00152	-.00133	-.02789	-.00180	-.00040	-.01027	-.00092
	E.S.D.	.00289	.02052	.00133	.00345	.02445	.00159	.00140	.00994	.00065
128	A.E.	.00061	-.00495	-.00117	.00072	-.00651	-.00141	.00039	-.00024	-.00070
	E.S.D.	.00204	.01464	.00096	.00242	.01738	.00114	.00099	.00713	.00047
256	A.E.	-.00074	-.01454	-.00001	-.00091	-.01756	.00000	-.00015	-.00711	-.00006
	E.S.D.	.00171	.01077	.00071	.00204	.01285	.00085	.00086	.00539	.00036
512	A.E.	.00157	-.00382	-.00007	.00188	-.00469	-.00009	.00084	-.00271	-.00006
	E.S.D.	.00119	.00765	.00051	.00142	.00912	.00061	.00060	.00386	.00026

A.E. -- Actual Error of Identified Values
E.S.D. -- Estimated Standard Deviation Using $\hat{\underline{R}}$

Both OS and XS bus power changes have RMS values 10% of nominal

Table 6.2(a) Identification Error For Increasing Measurements ($\underline{ZBB}$)

$\widehat{\underline{YEQX}} + \underline{YTL}$

Number of Measurement Sets	Element	Row 1: 1,1	Row 1: 1,2	Row 1: 1,3	Row 2: 2,1	Row 2: 2,2	Row 2: 2,3	Row 3: 3,1	Row 3: 3,2	Row 3: 3,3
	Actual Value	11.50	-6.14	-5.36	-6.14	14.26	-8.12	-5.36	-8.12	13.47
16	A.E.	-3.14	11.22	-8.23	-3.09	11.25	-8.51	-12.45	43.47	-31.33
32	A.E.	-0.84	2.57	-1.61	-0.83	3.12	-1.98	-4.81	16.01	-9.75
64	A.E.	-2.09	8.28	-4.52	-2.95	12.16	-6.99	-3.33	11.11	-3.49
128	A.E.	-0.70	1.58	-0.05	-1.13	2.91	-0.63	0.24	-4.27	6.22
256	A.E.	-0.87	4.34	-3.43	-1.15	5.97	-4.81	-2.15	7.71	-5.23
512	A.E.	-0.56	0.83	-0.35	-0.72	1.09	-0.46	1.84	3.88	-2.04

$(\widehat{\underline{YEQX}} + \underline{YTL})$ found from $\widehat{\underline{ZBB}}^{-1} - (\underline{YEQI} + \underline{YBQS})$

A.E. -- Actual Error of Identified Values

RMS values of QS and XS bus power changes 10% of nominal

Table 6.2(b) Identification Error for Increasing Measurements $(\underline{YEQX} + \underline{YTL})$

Errors in Predicting Line Power Flow Changes

Number of Measurement Sets	Line		1	2	3	4	5	6	7	8
	Nominal Line Power Flow		433.6	233.6	133.6	83.6	79.3	102.6	177.4	124.7
16		Change	11.818	8.860	5.906	5.304	2.148	2.523	33.363	1.424
		Error	.425	.425	.425	.425	.102	.102	3.023	.311
32		Change	16.700	8.984	5.511	4.732	2.893	2.864	31.612	1.912
		Error	.129	.129	.129	.129	.022	.022	1.096	.083
64		Change	12.220	7.236	5.795	5.356	3.356	3.107	28.477	1.773
		Error	.455	.455	.455	.455	.131	.131	2.740	.435
128		Change	11.596	7.550	4.200	5.885	2.885	2.403	27.892	1.936
		Error	.210	.211	.210	.210	.059	.059	1.416	.200
256		Change	13.832	6.589	4.639	5.189	2.466	2.028	25.938	1.456
		Error	.197	.197	.197	.197	.064	.064	1.068	.206
512		Change	9.226	8.122	6.316	5.782	2.703	3.026	27.274	1.697
		Error	.155	.154	.155	.155	.044	.044	.935	.146

Change -- RMS value of changes in line flow for 20 changes in bus power pattern in OS.

Error -- RMS value of error in predicting the changes using the identified model.

Changes in OS bus powers were generated by uniformly distributed random numbers such that OS powers varied up to ±10% of nominal values.

Table 6.2(c) Identification Error For Increasing Measurements (Line Flows)

$\underline{\widehat{YEQX}} + \underline{YTL}$

Actual Value	11.50	-6.14	-5.36
Error BBIM	.00	-1.37	1.30
Error TLPFM	.30	-14.44	10.66
E.S.D. TLPFM	.19	.40	.39
Actual Value	-6.14	14.26	-8.12
Error BBIM	.07	-1.87	2.97
Error TLPFM	4.52	-19.37	14.18
E.S.D. TLPFM	.24	.52	.51
Actual Value	-5.36	-8.12	13.47
Error BBIM	.52	- .52	2.79
Error TLPFM	5.41	-23.24	16.07
E.S.D. TLPFM	.60	1.29	1.26

Actual Value -- Actual Value of $\underline{YEQX} + \underline{YTL}$

Error BBIM -- Error in Identifying $\underline{\widehat{YEQX}} + \underline{YTL}$ using the Boundary Bus Impedance Model $\underline{\widehat{YEQX}} + \underline{YTL} = \underline{\widehat{ZBB}} - (\underline{YEQI} + \underline{YBOS})$

Error TLPFM -- Error in Identifying $\underline{\widehat{YEQX}} + \underline{YTL}$ using the Tie Line Power Flow Model

E.S.D. TLPFM -- Estimated Standard Deviation using $\underline{\hat{R}}$ from the Tie Line Power Flow Model

OS and XS varied equal amounts (10% of nominal) and the same 256 measurements used for both models.

Table 6.3 Comparison of Identification Models

$\underline{\hat{ZBB}}$ for 30 Bus System

Actual Value	.0644	.0489	.0491
Actual Error	-.0016	.0004	.0179
E.S.D.	.0019	.0020	.0217
Actual Value	.0489	.0665	.0663
Actual Error	-.0019	.0007	.0184
E.S.D.	.0021	.0023	.0244
Actual Value	.0491	.0663	.1122
Actual Error	-.0021	.0010	.0222
E.S.D.	.0025	.0026	.0282

E.S.D. -- Estimated Standard Deviation using $\underline{\hat{R}}$.

$\underline{\hat{ZBB}}$ Identified by the Boundary Bus Impedance Model

10% variation of bus powers in both XS and OS.

256 measurement sets used.

Table 6.4 Identification of a Lower Voltage Network

7. USE IN POWER SYSTEMS--SOME PRACTICAL ASPECTS

7.1 Use of Bus Power Inputs

From equation (4.10) which relates the error covariance of the estimate, it is obvious that the input directly affects the error. For the boundary bus impedance model $\underline{u}(n) = \underline{PEQI}(n) + \underline{PB}(n)$. As discussed previously and shown in Appendix C, $\underline{PEQI}(n)$ really depends on the operating point in a nonlinear manner so that it is possible for $\underline{PEQI}(n)$ to introduce significant error. The error from $\underline{PB}(n)$ on the other hand is only due to measurement noise. For this reason, it should turn out that the identification method presented will be most successful when the boundary buses have relatively large power injections (generators or loads) which supply much of the known variation. Or in other words, it would be nice to have good, strong input signals. In actuality IS may be of relatively large size so that contributions of $\underline{PI}(n)$ to $\underline{PEQI}(n) = \underline{AQI}\ \underline{PI}(n)$ from far across the network may be of questionable value and accuracy due to the linear approximation of $\underline{AQI}$. Since intuitively, buses in IS closest to BS will have the most affect on $\underline{PEQI}(n)$ and involve the least error, it may be practical to use only part of the contribution of $\underline{PI}(n)$ to $\underline{PEQI}(n)$ and lump the rest with $\underline{PEQX}(n)$.

7.2 Model Verification

Although contrary arguments could be made, the boundary bus impedance model is probably the better of the two models since it does not involve estimating a significant bias. However, the tie line power

flow model is still valid for verification purposes. For the boundary bus impedance model from equation (4.3)

$$\underline{\hat{R}} = \frac{1}{N}\sum_{n=1}^{N} c(n)\underline{\hat{v}}(n)\underline{\hat{v}}'(n)$$

$$= \frac{1}{N}\sum_{n=1}^{N} c(n)\widehat{\underline{ZBB}}\ \widehat{\underline{PEQX}}(n)\widehat{\underline{PEQX}}'(n)\widehat{\underline{ZBB}} \tag{7.1}$$

If $\underline{\hat{R}}$ is premultiplied and post multiplied by $\widehat{\underline{ZBB}}^{-1}$ then an estimate of the covariance of $\underline{PEQX}(n)$ is

$$\underline{\hat{R}}_{PEQX} = \widehat{\underline{ZBB}}^{-1}\underline{\hat{R}}\ \widehat{\underline{ZBB}}^{-1} \tag{7.2}$$

Now if measurements of $\underline{PTL}(n)$ were taken at the same times as measurements of $\underline{DB}(n)$ and $\underline{PEQI}(n) + \underline{PB}(n)$, and if the estimate of $\underline{YEQX}$ is

$$\widehat{\underline{YEQX}} = \widehat{\underline{ZBB}}^{-1} - \underline{YEQI} - \underline{YBB}$$

then equation (7.3) evaluated for $\widehat{\underline{YEQX}}$ should be close to $\underline{\hat{R}}_{PEQX}$ of equation (7.2).

$$\frac{1}{N}\sum_{n=1}^{N} c(n)\underline{PEQX}(n)\underline{PEQX}'(n)$$

$$= \frac{1}{N}\sum_{n=1}^{N}\Big\{\big[\underline{PTL}(n) - (\underline{YTL} + \underline{YEQX})\underline{DB}(n)\big]$$

$$\times\big[\underline{PTL}(n) - (\underline{YTL} + \underline{YEQX})\underline{DB}(n)\big]'\ c(n)\Big\} \tag{7.3}$$

If the two were not close, then the tie line power flow model would be indicating a lack of validity for the estimate of $\widehat{\underline{ZBB}}$ from the boundary bus impedance model. The tie line power flow model could also be used "on-line" once $\widehat{\underline{YEQX}}$ had been identified to provide a continuing check of the validity of the identified equivalent system. Should the measured value of $\underline{PTL}(n)$ vary from $(\underline{YTL} + \widehat{\underline{YEQX}})\underline{DB}(n)$ an amount not commensurate

with $\frac{1}{c(n)} \hat{\underline{R}}_{PEQX}$, that would indicate a change in XS had taken place. The change of course could either be in transmission line status affecting $\underline{YEQX}$ or a significant change in $\underline{PX}(n)$ affecting $\underline{PEQX}(n)$.

7.3 Estimating Equivalent Power Injections

Once $\widehat{\underline{YEQX}}$ has been identified, the actual values of $\underline{PEQX}(t_n)$ could be estimated. With enough measurements it might be possible to develop an approximate daily pattern for $\underline{PEQX}(t)$ so that they could be used as pseudo measurements [8] for the purpose of predicting a future power flow situation. If such were the case, the equivalent system could be extended in use from predicting changes in OS during which the change PEQX(n) would be small to use in predicting actual line power flows for given power injections in OS.

7.4 Inputs Which Decrease Identification Error

It has been assumed that the identification would be primarily passive in nature (i.e. by "listening" to the system) so that any power changes or voltage angle changes used to identify the equivalent system would be due to changes in the system's load demands. It was also argued that $\frac{1}{c(n)} \underline{R}$ was a reasonable approximation to the noise covariance where c(n) is inversely proportional to the time, $t_{n+1} - t_n$, between measurements. However, any large, relatively fast changes in OS would provide an input, output measurement set with relatively little noise ($\underline{PEQX}(n)$). These changes could be planned or accidental. If planned changes were used, it would be desirable to obtain a maximum of

invormation with a minimum of change. For this case it might be possible to formulate the problem in terms of minimizing a cost function subject to a constraint such as finding the inputs $\underline{u}(n)$ $(n=1,2,\ldots,N)$ which minimizes the cost

$$c = f_1(\text{error in } \hat{\underline{H}}) + f_2[\underline{u}(n)\underline{u}'(n), (n = 1,2,\ldots,N)]$$

subject to

$$\underline{z}(n) = \underline{H}\ \underline{u}(n) + \underline{v}(n) \qquad (n = 1,2,\ldots,N)$$

where f_1 and f_2 are scalar functions.

7.5 Use of the Equivalent System When the External System is Known

Although the linear load flow model was used to solve the problem of finding equivalent systems with no prior knowledge of the external systems, there is still an advantage to using the model when enough is known about the external system so that, for instance YEQX and PEQX could be found analytically for a particular system. The linear model clears the smoke so to speak, so that a good approximate picture can be easily obtained. For example AQX (and AQI) gives a general overview of how power is routed by the network while a comparison of YEQX with YBB shows at a glance the relative coupling between the two networks.

7.6 Identification of the Nonlinear Model

Use of the linear model may be sufficient to predict line power flows and voltage angles. However, in order to predict the effect of XS on voltage magnitudes within OS, the full nonlinear model is necessary. Developed in Appendix C are the equations relating complex bus voltages to complex bus power and current injections. The computation required and complexity of identifying the nonlinear equivalent model would likely be very much more than for the linear model. The increased accuracy in predicting real power flows would most likely be minimal, but the linear model cannot predict voltage changes due to the structure of XS.

8. CONCLUSIONS

8.1 Areas for Further Study

The tie line power flow model produces a biased estimate of the equivalent model parameters, however, if an unbiased estimate could be obtained so that accuracy was comparable to the boundary bus impedance model, there would probably be an overwhelming advantage to using the tie line power flow model. One of the most important reasons is that it uses variables from a local region only. $\underline{PTL}(t)$ is directly measurable and in order to find $\underline{DB}(t)$ state estimation would be required only in a small area around Boundary System. (Power coming in from the rest of Own System could be treated the same as $\underline{PTL}(t)$ for purposes of state estimation.) Also the linear tie line power flow model appears to be less of an approximation to its nonlinear analog (see Appendix C) than the linear boundary bus impedance model is to its nonlinear analog. Therefore, it is suggested that a study to find a method for eliminating bias in the tie line power flow model would be worthwhile.

The simulation carried out used a linear load flow model for the purpose of generating data used to identify the equivalent model. This simplified the simulation and it also served to eliminate errors which would be caused by variations from ideal conditions (low line resistance and voltage magnitude nearly constant). A more realistic simulation should be made to find how data from a nonlinear model affects accuracy of the identified linear model parameters. Also the net power flow summed over all the tie lines is usually held to a scheduled value. The effect of this on the identification should also be studied.

As mentioned previously, the identification accuracy could be

increased by varying the generation pattern for the purpose of probing the external system. However it would be necessary to maximize information and minimize the variation required. It is felt that one step in the proper direction might be to move one generator at a time since this would eliminate any interference among the inputs. A look at the boundary bus impedance model written in terms of the general model $\underline{z}(n) = \underline{H}\ \underline{u}(n) + \underline{v}(n)$ shows that if elements of $\underline{u}(n)$ were all zero except for element i, then the i^{th} column of $\underline{H}$ would be $\frac{1}{u_i(n)}\left[\underline{z}(n) - \underline{v}(n)\right]$. A study into this problem should result in a more sophisticated solution to a rather complex problem.

An obvious extension of identification of the linear model is the identification of the full nonlinear model. It is felt that the complexities involved would be an order of magnitude above that required for identification of the linear model. The accuracy gained in predicting voltage phase angle changes would probably be small, but voltage magnitude changes which have been ignored, would be available from a nonlinear model. If a search type of solution were used, then of course the solution to the linear model identification would give a starting point.

8.2 Summary

The problem of predicting how an electric power network will interact with other networks has been solved by reducing the entire outside world down to a relatively small equivalent model. The problem is complicated by the facts that (1.) no knowledge of parameters or variables of the outside world can be assumed, (2.) identification must be primarily passive, and (3.) "signal to noise ratios" can be expected to be about 1:1.

A linear model for the overall system was developed and its use justified with respect to the more conventional nonlinear model by considering accuracy versus computation requirements, insight, and complexity of the identification approach required. Two models were then proposed with each having certain advantages and shortcomings. The tie line power flow model uses data which is relatively accurate and locally available, but it produces a biased estimate. On the other hand the boundary bus impedance model produces an unbiased estimate but it requires data from all over the system and also requires manipulation of the equations for own system. It was argued that changes which occur in power and bus voltage angle between two times should be used with the hope of obtaining a zero mean disturbance and also to avoid the use of matrices whose elements have magnitudes very large compared to the determinant of the matrix. Since both models were in the general form of a linear, static, input-output relation with additive disturbances, a solution to the general problem was found under assumed conditions. Then the solution was applied to the two models and it was shown how the tie line power flow model produces a biased estimate. The simulation made to test the method was discussed showing the excellent correspondence between theory and simulation results. Finally, practical aspects of the problem were briefly discussed including model verification, estimation of the disturbances, system probing, and use of the linear model reduction method for purposes other than identification.

This method is proposed as a means of modeling the outside world and finding the parameters of that model.. The actual solution is in a very simple form and could probably be implemented as an extension of

state estimation techniques. It is by no means the ideal answer to how the outside world should be modeled, but it does allow a prediction of how the outside world will interact with the system of interest.

APPENDIX A. NONLINEAR LOAD FLOW EQUATIONS

For the pi transmission line model shown in Figure 2.1, the current flowing into the line from bus i is

$$\overline{IL}_{ik} = \bar{E}_i \, \overline{ys}_{ik} + (\bar{E}_i - \bar{E}_k) \, \bar{y}_{ik} \tag{A.1}$$

when the line is connected between buses i and k. $\overline{ys}_{ik}$ is the shunt capacitive admittance, $\bar{y}_{ik}$ is the series admittance, and $\bar{E}_i$ and $\bar{E}_k$ are the voltages at buses i and k respectively. All are complex quantities and

$$\overline{ys}_{ik} = j ys_{ik} = j\omega \frac{C_{ik}}{2} \tag{A.2}$$

$$\bar{y}_{ik} = y_{ik} \, e^{-j\phi_{ik}} = \frac{1}{R_{ik} + jX_{ik}} \tag{A.3}$$

$$\bar{E}_i = E_i \, e^{jD_i} \tag{A.4}$$

$$\bar{E}_k = E_k \, e^{jD_k} \tag{A.5}$$

where R_{ik}, X_{ik}, and ys_{ik} are the resistance, reactance, and capacitive susceptance of the transmission line between buses i and k. D_i and D_k are the voltage phase angles at buses i and k as measured with respect to a reference. Equation (A.1) can be rewritten

$$\overline{IL}_{ik} = \bar{E}_i (\overline{ys}_{ik} + \bar{y}_{ik}) - \bar{y}_{ik} \bar{E}_k \tag{A.6}$$

By Kirchhoff's current law, the sum of currents entering the transmission lines connected to bus i must equal the current being injected into the bus by generators or loads (positive for generators and negative for loads). $\bar{I}_i$ is the current injected into bus i.

$$\bar{I}_i = \sum_k \overline{IL}_{ik} = \bar{E}_i \sum_k (\overline{ys}_{ik} + \bar{y}_{ik}) - \sum_k \bar{y}_{ik}\bar{E}_k \tag{A.7}$$

Let $\bar{\underline{Y}}_{bus}$ be a complex bus admittance matrix whose diagonal element in row i is the sum of shunt and series admittances of all lines connected to bus i

$$(\bar{\underline{Y}}_{bus})_{ii} = \sum_k (\overline{ys}_{ik} + \bar{y}_{ik}) = \bar{Y}_{ii} = \bar{Y}_{ii}\ e^{-j\theta_{ii}} \tag{A.8}$$

and whose off diagonal ik and ki elements are the negative value of the series admittance of the line between buses i and k (or zero if there is no line between buses i and k).

$$(\bar{\underline{Y}}_{bus})_{ik} = -\bar{y}_{ik} = \bar{Y}_{ik} = Y_{ik}\ e^{-j\theta_{ik}} \tag{A.9}$$

$\bar{\underline{Y}}_{bus}$ is a symmetric matrix. Then for complex vectors $\bar{\underline{I}}_{bus}$ and $\bar{\underline{E}}_{bus}$ whose i^{th} elements are the current injected into bus i and the voltage at bus i respectively, equation (A.7) can be written in matrix form as

$$\bar{\underline{I}}_{bus} = \bar{\underline{Y}}_{bus}\ \bar{\underline{E}}_{bus} \tag{A.10}$$

When Kirchhoff's current law is applied to the entire network, it is obvious that for a system with K+1 buses, the current injections into K of the buses determine the current injected at the $(K+1)^{th}$ bus. Similarly, the voltage angles are relative to one another so one bus must be designated the reference bus for voltage angle whereas the reference for voltage magnitude is ground.

Real and reactive power injected into bus i, $P_i + jQ_i$, in terms of the current injected and bus voltage is

$$P_i + jQ_i = \bar{E}_i\ \bar{I}_i^* \tag{A.11}$$

or

$$P_i - jQ_i = \bar{E}_i^*\bar{I}_i \tag{A.12}$$

where $\bar{E}_i^*$ is the complex conjugate of voltage at bus i. Using equation (A.7) along with (A.8) and (A.9) where $\bar{Y}_{ik}$ is defined, equation (A.12) is

$$P_i - jQ_i = \bar{E}_i^* \sum_{k=1}^{K+1} \bar{E}_k \bar{Y}_{ik}$$

$$= E_i e^{-jD_i} \sum_{k=1}^{K+1} E_k e^{jD_k} Y_{ik} e^{-j\theta_{ik}} \tag{A.13}$$

$$P_i - jQ_i = \sum_{k=1}^{K+1} E_i E_k Y_{ik} e^{-j(\theta_{ik}+D_i-D_k)} \tag{A.14}$$

The load flow equation, (A.14), can be used for networks consisting of transmission lines and fixed tap transformers. However for other circumstances such as voltage controlled buses, tap changing transformers, and phase shifting transformers the equation must be modified. Stagg and El-Abiad [10] is one reference for these and other variations.

APPENDIX B. LINEAR LOAD FLOW EQUATIONS

The linear approximation for real power flow in a transmission line between bus i and bus k is

$$PL_{ik} = \tilde{E}_i \tilde{E}_k y_{ik}(D_i - D_k) \tag{B.1}$$

where D_i and D_k are the voltage phase angles at buses i and k as measured with respect to a reference, $\tilde{E}_i$ and $\tilde{E}_k$ are the nominal values of the bus voltage magnitudes, and y_{ik} is the transmission line admittance magnitude. This approximation is subject to the conditions

1. $D_i - D_k$ is small.
2. The bus voltage magnitudes do not vary much from their nominal values.
3. The ratio of transmission line resistance to reactance is small.

For convenience it will be assumed that $\tilde{E}_i = \tilde{E}_k = 1.0$ (or equivalently y_{ik} is normalized for $\tilde{E}_i$ and $\tilde{E}_k$) so that equation (B.1) is

$$PL_{ik} = y_{ik}(D_i - D_k) = D_i y_{ik} - y_{ik} D_k \tag{B.2}$$

Assuming no losses within the bus, by continuity of power the power being injected into a bus (positive for generators and negative for loads) is equal to the algebraic sum of the power flows into the transmission lines connected to that bus. Let P_i be the power injection at bus i, then

$$P_i = \sum_k PL_{ik} = D_i \sum_k y_{ik} - \sum_k y_{ik} D_k \tag{B.3}$$

Define $\underline{P}$ to be a vector whose i^{th} element is the power injected into bus i, and define $\underline{D}$ to be a vector of corresponding bus voltage angles.

Then relation (B.3) becomes

$$\underline{P} = \underline{Y}\,\underline{D} \tag{B.4}$$

By comparison with (B.3) it can be seen that the elements of $\underline{Y}$ are such that the diagonal element of row i is the sum of admittance magnitudes of the transmission lines which are connected to bus i, $Y_{ii} = \sum_k y_{ik}$. The off diagonal elements ik and ki are negative value of the admittance magnitude of the line between bus i and bus k, $Y_{ik} = Y_{ki} = -y_{ik}$. Therefore the matrix $\underline{Y}$ is of the form

$$\underline{Y} = \begin{bmatrix} \sum_k y_{1k} & -y_{12} & -y_{13} & --- \\ -y_{12} & \sum_k y_{2k} & -y_{23} & --- \\ -y_{13} & -y_{23} & \sum_k y_{3k} & --- \\ --- & --- & --- & --- \end{bmatrix} \tag{B.5}$$

Because each bus is usually only connected to a few of all the other buses, $\underline{Y}$ is normally a sparce matrix. It is readily observable that the sum of all elements of any row or column of $\underline{Y}$ is zero. The bus voltage phase angles must be measured with respect to a common reference so one bus must be designated as the reference bus where voltage angle is specified. Also, because the linear model is lossless, by continuity of power, the algebraic sum of elements of $\underline{P}$ is zero. Hence one row of equation (B.4) is redundant. If bus i is chosen as reference for voltage phase angle, then let $\underline{D}_{bus}$ be the vector $\underline{D}$ with element i deleted, let $\underline{P}_{bus}$ be the vector $\underline{P}$ with element i deleted, and let $\underline{Y}_{bus}$ be the matrix $\underline{Y}$ with row i and column i deleted. Then equation (B.4) is

$$\underline{P}_{bus} = \underline{Y}_{bus}\,\underline{D}_{bus} \tag{B.6}$$

$\underline{P}_{bus}$ and $\underline{D}_{bus}$ can be separated into vectors $\underline{PI}$ and $\underline{DI}$ for Internal System (IS), $\underline{PB}$ and $\underline{DB}$ for Boundary System (BS), and $\underline{PX}$ and $\underline{DX}$ for External System (XS). Similarly separating $\underline{Y}_{bus}$ into submatrices

$$\begin{bmatrix} \underline{PI} \\ \underline{PB} \\ \underline{PX} \end{bmatrix} = \begin{bmatrix} \underline{YII} & \underline{YIB} & \underline{YIX} \\ \underline{YIB}' & \underline{YBB} & \underline{YBX} \\ \underline{YIX}' & \underline{YBX}' & \underline{YXX} \end{bmatrix} \begin{bmatrix} \underline{DI} \\ \underline{DB} \\ \underline{DX} \end{bmatrix} \tag{B.7}$$

However by definition there are no transmission lines between buses of IS and XS so $\underline{YIX} = \underline{0}$.

$$\begin{bmatrix} \underline{PI} \\ \underline{PB} \\ \underline{PX} \end{bmatrix} = \begin{bmatrix} \underline{YII} & \underline{YIB} & \underline{0} \\ \underline{YIB}' & \underline{YBB} & \underline{YBX} \\ \underline{0} & \underline{YBX}' & \underline{YXX} \end{bmatrix} \begin{bmatrix} \underline{DI} \\ \underline{DB} \\ \underline{DX} \end{bmatrix} \tag{B.8}$$

The inverse of the bus admittance matrix $\underline{Y}_{bus}$ is the bus impedance matrix $\underline{Z}_{bus}$ such that

$$\underline{D}_{bus} = \underline{Z}_{bus}\ \underline{P}_{bus} \tag{B.9}$$

or

$$\begin{bmatrix} \underline{DI} \\ \underline{DB} \\ \underline{DX} \end{bmatrix} = \begin{bmatrix} \underline{ZII} & \underline{ZIB} & \underline{ZIX} \\ \underline{ZIB}' & \underline{ZBB} & \underline{ZBX} \\ \underline{ZIX}' & \underline{ZBX}' & \underline{ZXX} \end{bmatrix} \begin{bmatrix} \underline{PI} \\ \underline{PB} \\ \underline{PX} \end{bmatrix} \tag{B.10}$$

$\underline{Z}_{bus}$ in terms of submatrices of $\underline{Y}_{bus}$ can be found by solving $\underline{Z}_{bus}\ \underline{Y}_{bus} = \underline{I}$ for $\underline{Z}_{bus}$ where $\underline{I}$ is the identity matrix.

$$\begin{bmatrix} \underline{ZII} & \underline{ZIB} & \underline{ZIX} \\ \underline{ZIB}' & \underline{ZBB} & \underline{ZBX} \\ \underline{ZIX}' & \underline{ZBX}' & \underline{ZXX} \end{bmatrix} \begin{bmatrix} \underline{YII} & \underline{YIB} & \underline{0} \\ \underline{YIB}' & \underline{YBB} & \underline{YBX} \\ \underline{0} & \underline{YBX}' & \underline{YXX} \end{bmatrix} = \begin{bmatrix} \underline{I} & \underline{0} & \underline{0} \\ \underline{0} & \underline{I} & \underline{0} \\ \underline{0} & \underline{0} & \underline{I} \end{bmatrix} \tag{B.11}$$

Multiplying the middle row of $\underline{Z}_{bus}$ by the left column of $\underline{Y}_{bus}$ (here rows and columns will refer to row and column submatrices.)

$$\underline{ZIB}' \ \underline{YII} + \underline{ZBB} \ \underline{YIB}' = \underline{0} \tag{B.12}$$

and solving for $\underline{ZIB}'$

$$\underline{ZIB}' = -\underline{ZBB} \ \underline{YIB}' \ \underline{YII}^{-1} \tag{B.13}$$

Multiplying the middle row of $\underline{Z}_{bus}$ by the right column of $\underline{Y}_{bus}$ and solving for $\underline{ZBX}$

$$\underline{ZBB} \ \underline{YBX} + \underline{ZBX} \ \underline{YXX} = \underline{0} \tag{B.14}$$

$$\underline{ZBX} = -\underline{ZBB} \ \underline{YBX} \ \underline{YXX}^{-1} \tag{B.15}$$

Multiplying the middle row of $\underline{Z}_{bus}$ by the middle column of $\underline{Y}_{bus}$ and solving for $\underline{ZBB}$ with the use of (B.13) and (B.15)

$$\underline{ZIB}' \ \underline{YIB} + \underline{ZBB} \ \underline{YBB} + \underline{ZBX} \ \underline{YBX}' = \underline{I} \tag{B.16}$$

$$-\underline{ZBB} \ \underline{YIB}' \ \underline{YII}^{-1} \ \underline{YIB} + \underline{ZBB} \ \underline{YBB} - \underline{ZBB} \ \underline{YBX} \ \underline{YXX}^{-1}\underline{YBX}' = \underline{I} \tag{B.17}$$

$$\underline{ZBB} = (-\underline{YIB}' \ \underline{YII}^{-1} \ \underline{YIB} + \underline{YBB} - \underline{YBX} \ \underline{YXX}^{-1} \ \underline{YBX}')^{-1} \tag{B.18}$$

Multiplying the top row of $\underline{Z}_{bus}$ by the left column of $\underline{Y}_{bus}$ and solving for $\underline{ZII}$ with the use of (B.13)

$$\underline{ZII} \ \underline{YII} + \underline{ZIB} \ \underline{YIB}' = \underline{I} \tag{B.19}$$

$$\underline{ZII} = \underline{YII}^{-1} - \underline{ZIB} \ \underline{YIB}' \ \underline{YII}^{-1} \tag{B.20}$$

$$\underline{ZII} = \underline{YII}^{-1} + \underline{YII}^{-1} \ \underline{YIB} \ \underline{ZBB} \ \underline{YIB}' \ \underline{YII}^{-1} \tag{B.21}$$

Multiplying the top row of $\underline{Z}_{bus}$ by the right column of $\underline{Y}_{bus}$ and solving for ZIX with the use of (B.13) again

$$\underline{ZIB} \ \underline{YBX} + \underline{ZIX} \ \underline{YXX} = \underline{0} \tag{B.22}$$

$$\underline{ZIX} = - \underline{ZIB}\ \underline{YBX}\ \underline{YXX}^{-1} \tag{B.23}$$

$$\underline{ZIX} = \underline{YII}^{-1}\underline{YIB}\ \underline{ZBB}\ \underline{YBX}\ \underline{YXX}^{-1} \tag{B.24}$$

Let matrices $\underline{YEQI}$, $\underline{YEQX}$, $\underline{AQI}$, and $\underline{AQX}$ be defined by

$$\underline{YEQI} = -\underline{YIB}'\ \underline{YII}^{-1}\ \underline{YIB} \tag{B.25}$$

$$\underline{YEQX} = -\underline{YBX}\ \underline{YXX}^{-1}\ \underline{YBX}' \tag{B.26}$$

$$\underline{AQI} = -\underline{YIB}'\ \underline{YII}^{-1} \tag{B.27}$$

$$\underline{AQX} = -\underline{YBX}\ \underline{YXX}^{-1} \tag{B.28}$$

Then equations for $\underline{ZBB}$, $\underline{ZII}$, $\underline{ZIB}$, $\underline{ZIX}$, and $\underline{ZBX}$ can be written

$$\underline{ZBB} = (\underline{YEQI} + \underline{YBB} + \underline{YEQX})^{-1} \tag{B.29}$$

$$\underline{ZII} = \underline{YII}^{-1} + \underline{AQI}'\ \underline{ZBB}\ \underline{AQI} \tag{B.30}$$

$$\underline{ZIB} = \underline{AQI}'\ \underline{ZBB} \tag{B.31}$$

$$\underline{ZIX} = \underline{AQI}'\ \underline{ZBB}\ \underline{AQX} \tag{B.32}$$

$$\underline{ZBX} = \underline{ZBB}\ \underline{AQX} \tag{B.33}$$

Using (B.10), the equations for $\underline{DI}$ and $\underline{DB}$ are

$$\begin{bmatrix} \underline{DI} \\ \\ \underline{DB} \end{bmatrix} = \begin{bmatrix} \underline{ZII} & \underline{ZIB} \\ \\ \underline{ZIB}' & \underline{ZBB} \end{bmatrix} \begin{bmatrix} \underline{PI} \\ \\ \underline{PB} \end{bmatrix} + \begin{bmatrix} \underline{ZIX} \\ \\ \underline{ZBX} \end{bmatrix} \underline{PX} \tag{B.34}$$

or using (B.29) through (B.30)

$$\begin{bmatrix} \underline{DI} \\ \\ \underline{DB} \end{bmatrix} = \left\{ \begin{bmatrix} \underline{YII}^{-1} & \underline{0} \\ \\ \underline{0} & \underline{0} \end{bmatrix} + \begin{bmatrix} \underline{AQI}' \\ \\ \underline{I} \end{bmatrix} \; \underline{ZBB} \begin{bmatrix} \underline{AQI} & \underline{I} \end{bmatrix} \right\} \begin{bmatrix} \underline{PI} \\ \\ \underline{PB} \end{bmatrix}$$

$$+ \begin{bmatrix} \underline{AQI}' \\ \\ \underline{I} \end{bmatrix} \; \underline{ZBB} \; \underline{AQX} \; \underline{PX} \tag{B.35}$$

From equation (B.35) the key role of ZBB is obvious.

APPENDIX C. NONLINEAR MODEL IDENTIFICATION

To avoid notation clutter in this appendix, all voltage and current vectors and admittance matrices will be understood to be complex vectors and matrices even though there is no bar over the symbol (for a complex quantity) or bar under the symbol (for a vector or matrix). For example E and I are complex vectors.

The complex matrix equation for relating bus voltages to bus current injections is

$$I_{bus} = Y_{bus}\, E_{bus} \tag{C.1}$$

I_{bus} and E_{bus} can be divided into separate vectors for Internal System (II and EI respectively), for Boundary System (IB and EB), and for External System (IX and EX). Dividing Y_{bus} into the corresponding submatrices and noting that by definition of IS, BS, and XS there is no immediate coupling between IS and XS, equation (C.1) can be written

$$\begin{bmatrix} II \\ IB \\ IX \end{bmatrix} = \begin{bmatrix} YII & YIB & 0 \\ YIB' & YBB & YBX \\ 0 & YBX' & YXX \end{bmatrix} \begin{bmatrix} EI \\ EB \\ EX \end{bmatrix} \tag{C.2}$$

The only coupling between CS and XS is due to YBB which can be separated into two matrices. That belonging to CS will be designated YBCS, and that belonging to XS will be designated YTL, so that YBB = YBCS + YTL. Separating equation (C.2)

$$\begin{bmatrix} II \\ IB \\ IX \end{bmatrix} = \begin{bmatrix} YII & YIB & 0 \\ YIB' & YBCS & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} EI \\ EB \\ EX \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & YTL & YBX \\ 0 & YBX' & YXX \end{bmatrix} \begin{bmatrix} EI \\ EB \\ EX \end{bmatrix} \qquad (C.3)$$

Removing the equation for CS

$$\begin{bmatrix} II \\ IB \end{bmatrix} = \begin{bmatrix} YII & YIB \\ YIB' & YBCS \end{bmatrix} \begin{bmatrix} EI \\ EB \end{bmatrix} + \begin{bmatrix} 0 \\ YTL\ EB + YBX\ EX \end{bmatrix} \qquad (C.4)$$

The quantity YTL EB + YBX EX is a vector of line current leaving the buses for BS over the tie lines between BS and XS. Designating this the tie line vector ITL

$$\begin{bmatrix} II \\ IB - ITL \end{bmatrix} = \begin{bmatrix} YII & YIB \\ YIB' & YBCS \end{bmatrix} \begin{bmatrix} EI \\ EB \end{bmatrix} \qquad (C.5)$$

$$ITL = YTL\ EB + YBX\ EX \qquad (C.6)$$

Removing the bottom row from equation (C.3)

$$IX = YBX'\ EB + YXX\ EX \qquad (C.7)$$

and solving for EX

$$EX = -YXX^{-1}\ YBX'\ EB + YXX^{-1}\ IX \qquad (C.8)$$

Substituting equation (C.8) into (C.6)

$$ITL = YTL\ EB - YBX\ YXX^{-1}\ YBX'\ EB + YBX\ YXX^{-1}\ IX \qquad (C.9)$$

Define an equivalent external bus admittance matrix YEQX and an equivalent external complex bus current injection vector IEQX by

$$YEQX = -YBX\ YXX^{-1}\ YBX' \tag{C.10}$$

$$IEQX = AQX\ IX \tag{C.11}$$

where $$AQX = -YBX\ YXX^{-1} \tag{C.12}$$

Then equation (C.6) becomes

$$ITL = (YTL + YEQX)EB - IEQX \tag{C.13}$$

and if equation (C.13) is substituted into equation (C.5), since $YBB = YBOS + YTL$

$$\begin{bmatrix} II \\ \\ IBB + IEQI \end{bmatrix} = \begin{bmatrix} YII & YIB \\ \\ YIB' & (YBB+YEQX) \end{bmatrix} \begin{bmatrix} EI \\ \\ EB \end{bmatrix} \tag{C.14}$$

In the same way, an equivalent complex bus admittance matrix and an equivalent complex bus current injection vector for IS as seen by BS can be found. The top row of equation (C.14) is

$$II = YII\ EI + YIB\ EB \tag{C.15}$$

Solving for EI

$$EI = -YII^{-1}\ YIB\ EB + YII^{-1}\ II \tag{C.16}$$

and substituting into the bottom row of equation (C.14)

$$IB + IEQI = -YIB'\ YII^{-1}\ YIB\ EB + YIB'\ YII^{-1}\ II + (YBB + YEQX)EB \tag{C.17}$$

Defining an equivalent internal complex bus admittance matrix YEQI, and an equivalent internal complex bus current injection vector IEQI by

$$YEQI = -YIB'\ YII^{-1}\ YIB \tag{C.18}$$

$$IEQI = AQI\ II \tag{C.19}$$

where $$AQI = -YIB'\ YII^{-1} \tag{C.20}$$

equation (C.17) becomes

$$IEQI + IB + IEQX = (YEQI + YBB + YEQX)EB \tag{C.21}$$

Just as YBB was divided into YTL and YBCS, so also YBCS can be divided into that belonging to IS designated as YBBS, with YBCS = YBIS + YBBS. Then equation (C.21) is

$$IEQI + IB + IEQX = (YBIS + YEQI)EB + YBBS\ EB + (YTL + YEQX)EB \tag{C.22}$$

Thus far equations (C.5), (C.13), (C.14), and (C.21) are the pertinent equations in terms of bus current injections. To place them in terms of bus power injections let a diagonal matrix whose elements are the complex conjugate values of bus voltages be

$$[E^*] \qquad \begin{bmatrix} E_1^* & 0 & 0 & --- \\ 0 & E_2^* & 0 & --- \\ 0 & 0 & E_3^* & --- \\ --- & --- & --- & --- \end{bmatrix}$$

(Note that [E] is a diagonal matrix whose diagonal elements are identical to the elements of the vector E.) Also, let PI and QI, PB and QB, and PX and QX be the real and reactive bus power injections of IS, BS, and XS. Then because for any bus i, $P_i - jQ_i = E_i^* I_i$

$$PI - jQI = [EI^*]\ II \tag{C.23}$$

$$PB - jQB = [EB^*]\ IB \tag{C.24}$$

$$PX - jQX = [EX^*]\ IX \tag{C.25}$$

$$PTL - jQTL = [EB^*]\ ITL \tag{C.25a}$$

or

$$II = [EI^*]^{-1}(PI - jQI) \tag{C.26}$$

$$IX = [EX^*]^{-1}(PX - jQX) \tag{C.27}$$

Multiplying equation (C.21) by $[EB^*]$

$$[EB^*](IEQI + IB + IEQX) = [EB^*](YEQI + YBB + YEQX)EB \tag{C.28}$$

Define equivalent internal and external bus power injection vectors by

$$PEQI - jQEQI = [EB^*]\ IEQI = [EB^*]\ AQI[EI^*]^{-1}(PI - jQI) \tag{C.28}$$

$$PEQX - jQEQX = [EB^*]\ IEQX = [EB^*]\ AQX[EX^*]^{-1}(PX - jQX) \tag{C.30}$$

Then (C.28) becomes

$$(PEQI - jQEQI) + (PB - jQB) + (PEQX - jQEQX) = [EB^*](YEQI + YBB + YEQX)EB \tag{C.31}$$

SUMMARY

For the following a bar over a symbol denotes a complex quantity while matrices and vectors are denoted by underlined symbols. Using equations (C.5), (C.23), (C.24), and (C.25a) the nonlinear load flow equation for CS is

$$\begin{bmatrix} \underline{PI} - j\underline{QI} \\ \\ (\underline{PB} - j\underline{QB}) - (\underline{PTL} - j\underline{QTL}) \end{bmatrix} = \begin{bmatrix} [\overline{\underline{EI}}^*] & \underline{0} \\ \\ \underline{0} & [\overline{\underline{EB}}^*] \end{bmatrix} \begin{bmatrix} \overline{\underline{YII}} & \overline{\underline{YIB}} \\ \\ \overline{\underline{YIB}}' & \overline{\underline{YBCS}} \end{bmatrix} \begin{bmatrix} \overline{\underline{EI}} \\ \\ \overline{\underline{EB}} \end{bmatrix} \tag{C.32}$$

which, if $\overline{\underline{YEQX}}$ and $\underline{PEQX} - j\underline{QEQX}$ are known, becomes

$$\begin{bmatrix} \underline{PI} - j\underline{QI} \\ (\underline{PB} - j\underline{QB}) + (\underline{PEQX} - j\underline{QEQX}) \end{bmatrix} = \begin{bmatrix} [\overline{EI}^{*}] & \underline{0} \\ \underline{0} & [\overline{EB}^{*}] \end{bmatrix} \begin{bmatrix} \overline{\underline{YII}} & \overline{\underline{YIB}} \\ \overline{\underline{YIB}}' & (\overline{\underline{YBB}}+\overline{\underline{YEQX}}) \end{bmatrix} \begin{bmatrix} \overline{\underline{EI}} \\ \overline{\underline{EB}} \end{bmatrix} \tag{C.33}$$

The nonlinear analog of the linear load flow tie line power flow model for identifying $\underline{YEQX}$ is

$$\underline{PTL} - j\underline{QTL} = [\overline{EB}^{*}]\ (\overline{\underline{YTL}} + \overline{\underline{YEQX}})\overline{\underline{EB}} - (\underline{PEQX} - j\underline{QEQX}) \tag{C.34}$$

and the nonlinear analog of the linear boundary bus impedance model is

$$[(\underline{PEQI} - j\underline{QEQI}) + (\underline{PB} - j\underline{QB})] + (\underline{PEQX} - j\underline{QEQX})$$

$$= [\overline{EB}^{*}][\overline{\underline{YEQI}} + \overline{\underline{YBB}} + \overline{\underline{YEQX}}]\ \overline{\underline{EB}} \tag{C.35}$$

Repeating the equations for current injections

$$\begin{bmatrix} \overline{\underline{II}} \\ \overline{\underline{IB}} - \overline{\underline{ITL}} \end{bmatrix} = \begin{bmatrix} \overline{\underline{YII}} & \overline{\underline{YIB}} \\ \overline{\underline{YIB}}' & \overline{\underline{YBB}} \end{bmatrix} \begin{bmatrix} \overline{\underline{EI}} \\ \overline{\underline{EB}} \end{bmatrix} \tag{C.36}$$

$$\begin{bmatrix} \overline{\underline{II}} \\ \overline{\underline{IB}} + \overline{\underline{IEQX}} \end{bmatrix} = \begin{bmatrix} \overline{\underline{YII}} & \overline{\underline{YIB}} \\ \overline{\underline{YIB}}' & (\overline{\underline{YBB}}+\overline{\underline{YEQX}}) \end{bmatrix} \begin{bmatrix} \overline{\underline{EI}} \\ \overline{\underline{EB}} \end{bmatrix} \tag{C.37}$$

$$\overline{\underline{ITL}} = (\overline{\underline{YTL}} + \overline{\underline{YEQX}})\overline{\underline{EB}} - \overline{\underline{IEQX}} \tag{C.38}$$

$$(\overline{\underline{IEQI}} + \overline{\underline{IB}}) + \overline{\underline{IEQX}} = (\overline{\underline{YEQI}} + \overline{\underline{YBB}} + \overline{\underline{YEQX}})\overline{\underline{EB}} \tag{C.39}$$

APPENDIX D. MAXIMUM LIKELIHOOD IDENTIFICATION EQUATIONS

Given a sequence of input measurements $\underline{u}(1), \underline{u}(2), \ldots, \underline{u}(N)$, a sequence of corresponding output measurements $\underline{z}(1), \underline{z}(2), \ldots, \underline{z}(N)$, a set of confidence coefficients $c(1), c(2), \ldots, c(N)$ which indicate the relative confidence associated with each input/output set, and given the input/output relation

$$\underline{z}(n) = \underline{H}\ \underline{u}(n) + \underline{v}(n) \tag{D.1}$$

where $\underline{v}(n)$ is a noise vector having a Gaussian probability distribution with zero mean and covariance $\frac{1}{c(n)}\underline{R}$, it is desired to find the maximum likelihood estimates of $\underline{H}$ and $\underline{R}$ subject to the following conditions:

1. The elements of $\underline{H}$ are independent of one another. (e.g. in general $\underline{H} \neq \underline{H}'$)
2. $E\left\{\underline{v}(n)\right\} = 0$
3. $E\left\{\underline{v}(n)\underline{v}'(m)\right\} = 0$ for $n \neq m$
4. $E\left\{\underline{v}(n)\underline{v}'(n)\right\} = \frac{1}{c(n)}\underline{R}$
5. $E\left\{\underline{v}(n)\ \underline{u}'(n)\right\} = 0$

The probability distribution function for the Kx1 vector $\underline{v}(n)$ is

$$p[\underline{v}(n)] = \left[(2\pi)^K c^{-K}(n)|\underline{R}|\right]^{-\frac{1}{2}} e^{-\frac{1}{2}\underline{v}'(n)c(n)\underline{R}^{-1}\underline{v}(n)} \tag{D.2}$$

The likelihood function for $\underline{z}(n)$ and $\underline{u}(n)$ given $\underline{H}$ and $\underline{R}$ where $\underline{z}(n)$ and $\underline{u}(n)$ are Kx1 vectors, and $\underline{H}$ and $\underline{R}$ are KxK matrices is

$$p[\underline{z}(n),\underline{u}(n):\underline{H},\underline{R}] = \left[(2\pi)^K c^{-K}(n)|\underline{R}|\right]^{-\frac{1}{2}} e^{J[\underline{z}(n),\underline{u}(n):\underline{H},\underline{R}]} \tag{D.3}$$

where

$$J[\underline{z}(n),\underline{u}(n):\underline{H},\underline{R}] = \frac{1}{2}[\underline{z}(n) - \underline{H}\ \underline{u}(n)]\, c(n)\underline{R}^{-1}[\underline{z}(n) - \underline{H}\ \underline{u}(n)]' \qquad (D.4)$$

Using $tr\{\underline{A}\}$ to mean the trace of matrix $\underline{A}$ and using the identity $\underline{a}'\underline{a} = tr\{\underline{a}\ \underline{a}'\}$ where $\underline{a}$ is a vector equation (D.4) can also be written

$$J[\underline{z}(n),\underline{u}(n):\underline{H},\underline{R}] = \frac{1}{2}\, tr\left\{c(n)\underline{R}^{-1}[\underline{z}(n) - \underline{H}\ \underline{u}(n)][\underline{z}(n) - \underline{H}\ \underline{u}(n)]'\right\} \qquad (D.5)$$

Let the sets $\underline{Z}$, $\underline{U}$, and $\underline{V}$ be such that $\underline{Z} = \{\underline{z}(1), \underline{z}(2), \ldots, \underline{z}(N)\}$, $\underline{U} = \{\underline{u}(1), \underline{u}(2), \ldots, \underline{u}(N)\}$, and $\underline{V} = \{\underline{v}(1), \underline{v}(2), \ldots, \underline{v}(N)\}$. Because $\underline{v}(n)$ is assumed to be uncorrelated in time $(E\{\underline{v}(n)\underline{v}'(m)\} = 0$ for $n \neq m)$ and because $\underline{v}(n)$ is Gaussian, the joint probability distribution function for $\underline{V}$ is the product of the distribution functions for $\underline{v}(1), \underline{v}(2), \ldots, \underline{v}(N)$.

$$p(\underline{V}) = p[\underline{v}(1)]\, p[\underline{v}(2)] \ldots p[\underline{v}(N)]$$

$$= \left[(2\pi)^{NK} |\underline{R}|^{N} \prod_{n=1}^{N} c^{-K}(n)\right]^{-\frac{1}{2}} e^{-\frac{1}{2}\sum_{n=1}^{N} \underline{v}'(n)c(n)\underline{R}^{-1}\underline{v}(n)} \qquad (D.6)$$

The likelihood function of $\underline{Z}$ and $\underline{U}$ given $\underline{H}$ and $\underline{R}$ is then

$$p(\underline{Z},\underline{U}:\underline{H},\underline{R}) = \left[(2\pi)^{NK} |\underline{R}|^{N} \prod_{n=1}^{N} c^{-K}(n)\right]^{-\frac{1}{2}} e^{-J(\underline{Z},\underline{U}:\underline{H},\underline{R})} \qquad (D.7)$$

where

$$J(\underline{Z},\underline{U}:\underline{H},\underline{R}) = \frac{1}{2}\sum_{n=1}^{N} tr\left\{c(n)\underline{R}^{-1}[\underline{z}(n) - \underline{H}\ \underline{u}(n)][\underline{z}(n) - \underline{H}\ \underline{u}(n)]'\right\} \qquad (D.8)$$

It is desired to find the values of $\underline{H}$ and $\underline{R}$ which maximize the likelihood function $p(\underline{Z},\underline{U}:\underline{H},\underline{R})$. These values of $\underline{H}$ and $\underline{R}$ are the maximum likelihood estimates $\hat{\underline{H}}$ and $\hat{\underline{R}}$. If the logarithm of the likelihood function is maximized, then so will the function. The log likelihood function is

$$\ell n[p(\underline{Z},\underline{U}:\underline{H},\underline{R})] = -\frac{1}{2} NK\, \ell n(2\pi) + \frac{1}{2} \sum_{n=1}^{N} K\, \ell n\, c(n)$$

$$- \frac{1}{2} N\, \ell n|\underline{R}| - J(\underline{Z},\underline{U}:\underline{H},\underline{R}) \qquad \text{(D.9)}$$

A new function can be defined

$$f(\underline{Z},\underline{U}:\underline{H},\underline{R}) = -2\ell n[p(\underline{Z},\underline{U}:\underline{H},\underline{R})] - NK\, \ell n(2\pi) + \sum_{n=1}^{N} K\, \ell n\, c(n)$$

$$= N\, \ell n|\underline{R}| + 2J(\underline{Z},\underline{U}:\underline{H},\underline{R}) \qquad \text{(D.10)}$$

$f(\underline{Z},\underline{U}:\underline{H},\underline{R})$ only has terms which involve $\underline{H}$ or $\underline{R}$, hence, minimizing $f(\underline{Z},\underline{U}:\underline{H},\underline{R})$ is equivalent to maximizing the likelihood function.

To perform the necessary mathematical manipulations certain matrix gradient identities are used from Athans and Schweppe [1]. The matrix gradient for a scalar function of a KxM matrix $\underline{X}$ is defined for use here as

$$\frac{\partial}{\partial \underline{X}} [f(\underline{X})] = \begin{bmatrix} \dfrac{\partial f(\underline{X})}{\partial X_{11}} & \cdots & \dfrac{\partial f(\underline{X})}{\partial X_{K1}} \\ - - - & - - - & - - - \\ \dfrac{\partial f(\underline{X})}{\partial X_{1M}} & \cdots & \dfrac{\partial f(\underline{X})}{\partial X_{KM}} \end{bmatrix} \qquad \text{(D.11)}$$

Two properties of trace of a matrix are

$$\mathrm{tr}\left\{\underline{A}\right\} = \mathrm{tr}\left\{\underline{A}'\right\} \qquad \text{(D.12)}$$

$$\mathrm{tr}\left\{\underline{A}\ \underline{B}\right\} = \mathrm{tr}\left\{\underline{B}\ \underline{A}\right\}$$

The identities used are

$$\frac{\partial}{\partial \underline{X}} \mathrm{tr}\left\{\underline{A}\ \underline{X}\right\} = \underline{A} \qquad \text{(D.14)}$$

$$\frac{\partial}{\partial \underline{X}} \operatorname{tr}\left\{\underline{A}\ \underline{X}\ \underline{B}\ \underline{X}'\right\} = \underline{B}\ \underline{X}'\ \underline{A} + \underline{B}'\ \underline{X}'\ \underline{A}' \tag{D.15}$$

$$\frac{\partial}{\partial \underline{X}} \operatorname{tr}\left\{\underline{A}\ \underline{X}^{-1}\ \underline{B}\right\} = -\underline{X}^{-1}\ \underline{B}\ \underline{A}\ \underline{X}^{-1} \tag{D.16}$$

$$\frac{\partial}{\partial \underline{X}} \ell n|\underline{X}| = \underline{X}^{-1} \tag{D.17}$$

When $f(\underline{Z},\underline{U},\underline{H},\underline{R})$ is at its minimum value, the matrix gradients of $f(\underline{Z},U:\underline{H},\underline{R})$ with respect to $\underline{H}$ and $\underline{R}$ will be zero. Then the maximum likelihood estimates $\hat{\underline{H}}$ and $\hat{\underline{R}}$ will be the solutions for $\underline{H}$ and $\underline{R}$ of

$$\frac{\partial}{\partial \underline{H}} f(\underline{Z},\underline{U}:\underline{H},\underline{R}) = \underline{0} \tag{D.18}$$

and

$$\frac{\partial}{\partial \underline{R}} f(\underline{Z},\underline{U}:\underline{H},\underline{R}) = \underline{0} \tag{D.19}$$

for

$$f(\underline{Z},\underline{U}:\underline{H},\underline{R}) = N\, \ell n|\underline{R}| + 2J(\underline{Z},\underline{U}:\underline{H},\underline{R}) \tag{D.20}$$

and

$$J(\underline{Z},\underline{U}:\underline{H},\underline{R}) = \frac{1}{2} \sum_{n=1}^{N} \operatorname{tr}\left\{c(n)\underline{R}^{-1}[\underline{z}(n) - \underline{H}\ \underline{u}(n)][\underline{z}(n) - \underline{H}\ \underline{u}(n)]'\right\} \tag{D.21}$$

First, the gradient with respect to $\underline{H}$ is

$$\frac{\partial}{\partial \underline{H}} f(\underline{Z},\underline{U}:\underline{H},\underline{R}) = \frac{\partial}{\partial \underline{H}} 2J(\underline{Z},\underline{U}:\underline{H},\underline{R}) = \underline{0}$$

$$= \frac{\partial}{\partial \underline{H}} \sum_{n=1}^{N} \operatorname{tr}\left\{c(n)\underline{R}^{-1}[\underline{z}(n) - \underline{H}\ \underline{u}(n)][\underline{z}(n) - \underline{H}\ \underline{u}(n)]'\right\}$$

$$= \sum_{n=1}^{N} \frac{\partial}{\partial \underline{H}} \operatorname{tr}\left\{c(n)\underline{R}^{-1}\underline{z}(n)\underline{z}'(n) - c(n)\underline{R}^{-1}\underline{z}(n)\underline{u}'(n)\underline{H}'\right.$$

$$\left. - c(n)\underline{R}^{-1}\underline{H}\ \underline{u}(n)\underline{z}'(n) + c(n)\underline{R}^{-1}\underline{H}\ \underline{u}(n)\underline{u}'(n)\underline{H}'\right\} \tag{D.22}$$

Using equations (D.12) and (D.13)

$$\mathrm{tr}\left\{c(n)\underline{R}^{-1}\underline{z}(n)\underline{u}'(n)\underline{H}'\right\} = \mathrm{tr}\left\{c(n)\underline{H}\ \underline{u}(n)\underline{z}'(n)\underline{R}^{-1}\right\}$$

$$= \mathrm{tr}\left\{c(n)\underline{u}(n)\underline{z}'(n)\underline{R}^{-1}\underline{H}\right\} \qquad (D.23)$$

$$\mathrm{tr}\left\{c(n)\underline{R}^{-1}\underline{H}\ \underline{u}(n)\underline{z}'(n)\right\} = \mathrm{tr}\left\{c(n)\underline{u}(n)\underline{z}'(n)\underline{R}^{-1}\underline{H}\right\} \qquad (D.24)$$

Then equation (D.22) is

$$\underline{0} = \sum_{n=1}^{N} \frac{\partial}{\partial \underline{H}}\ \mathrm{tr}\left\{-2c(n)\underline{u}(n)\underline{z}'(n)\underline{R}^{-1}\underline{H} + c(n)\underline{R}^{-1}\underline{H}\ \underline{u}(n)\underline{u}'(n)\underline{H}'\right\} \qquad (D.25)$$

and using equations (D.14) and (D.15)

$$\underline{0} = \sum_{n=1}^{N} \left[-2c(n)\underline{u}(n)\underline{z}'(n)\underline{R}^{-1} + 2c(n)\underline{u}(n)\underline{u}'(n)\underline{H}'\underline{R}^{-1}\right]$$

$$= 2\left[-\sum_{n=1}^{N} c(n)\underline{u}(n)\underline{z}'(n) + \sum_{n=1}^{N} c(n)\underline{u}(n)\underline{u}'(n)\underline{H}'\right]\underline{R}^{-1} \qquad (D.26)$$

Then unless $\underline{R}^{-1} = \underline{0}$ (which would mean infinite noise covariance)

$$\sum_{n=1}^{N} c(n)\underline{u}(n)\underline{z}'(n) = \sum_{n=1}^{N} c(n)\underline{u}(n)\underline{u}'(n)\underline{H}' \qquad (D.27)$$

$$\underline{H}\sum_{n=1}^{N} c(n)\underline{u}(n)\underline{u}'(n) = \sum_{n=1}^{N} c(n)\underline{z}(n)\underline{u}'(n) \qquad (D.28)$$

and the value of $\underline{H}$ which maximizes $p(\underline{Z},\underline{U}:\underline{H},\underline{R})$ is

$$\hat{\underline{H}} = \left[\sum_{n=1}^{N} c(n)\underline{z}(n)\underline{u}'(n)\right]\left[\sum_{n=1}^{N} c(n)\underline{u}(n)\underline{u}'(n)\right]^{-1} \qquad (D.29)$$

Taking the matrix gradient of $f(\underline{Z},\underline{U}:\underline{H},\underline{R})$ with respect to $\underline{R}$

$$\frac{\partial}{\partial \underline{R}} f(\underline{Z},\underline{U}:\underline{H},\underline{R}) = 0$$

$$= N \frac{\partial}{\partial \underline{R}} \ln|\underline{R}| + \frac{\partial}{\partial \underline{R}} \sum_{n=1}^{N} \mathrm{tr}\left\{c(n)\underline{R}^{-1}[\underline{z}(n) - \underline{H}\ \underline{u}(n)][\underline{z}(n) - \underline{H}\ \underline{u}(n)]'\right\} \tag{D.30}$$

Using equations (D.16) and (D.17)

$$\underline{0} = N\underline{R}^{-1} - \sum_{n=1}^{N} c(n)\underline{R}^{-1}[\underline{z}(n) - \underline{H}\ \underline{u}(n)][\underline{z}(n) - \underline{H}\ \underline{u}(n)]'\ \underline{R}^{-1}$$

$$= N\underline{R}^{-1}\left\{\underline{R} - \frac{1}{N}\sum_{n=1}^{N} c(n)[\underline{z}(n) - \underline{H}\ \underline{u}(n)][\underline{z}(n) - \underline{H}\ \underline{u}(n)]'\right\}\underline{R}^{-1} \tag{D.31}$$

Again unless $\underline{R}^{-1} = \underline{0}$, using equation (D.29) the value of $\underline{R}$ which minimizes $p(\underline{Z},\underline{U}:\underline{H},\underline{R})$ is

$$\hat{\underline{R}} = \frac{1}{N}\sum_{n=1}^{N} c(n)[\underline{z}(n) - \hat{\underline{H}}\ \underline{u}(n)][\underline{z}(n) - \hat{\underline{H}}\ \underline{u}(n)]' \tag{D.32}$$

Hence $\hat{\underline{H}}$ and $\hat{\underline{R}}$ or equations (D.29) and (D.32) are the maximum likelihood solutions to the problem.

APPENDIX E. COMPUTER SIMULATION PROGRAM

```
C  MAIN PROGRAM
C
C
C          COMPUTER      EXPLANATION AND SYMBOL USED IN TEXT
C          PROGRAM
C          SYMBOL
C
C          ZWS         IMPEDANCE MATRIX FOR WHOLE SYSTEM
C          YL          YL   LINE ADMITTANCE
C          KHEAD       BUS NUMBER OF BUS TO WHICH HEAD OF LINE IS CONNECTED
C          KTAIL       BUS NUMBER OF BUS TO WHICH TAIL OF LINE IS CONNECTED
C          P           P   VECTOR OF CHANGES IN BUS POWER INJECTIONS FOR
C                      WHOLE SYSTEM
C          PL          PL   LINE POWER FLOWS
C          D           D   BUS VOLTAGE ANGLE VECTOR
C          PNOM        PNOM   VECTOR OF NOMINAL VALUES OF BUS POWER INJECTIONS
C          ABOS        ( AOI I )  AOI WITH AN ADJOINED IDENTITY MATRIX
C          YIINV       INVERSE OF YII
C          ZOSEQ       ESTIMATED OWN SYSTEM IMPEDANCE MATRIX
C                      ZOSEQ = ( AOI I )'ZBBHAT( AOI I )  ZBBHAT IS ESTIMATED ZBB
C          Z           Z   OUTPUT VECTOR
C          X           U   INPUT VECTOR
C          V           V   DISTURBANCE VECTOR
C          CHI         ACCUMULATED SUM OF Z*X
C          SGINV       ACCUMULATED SUM OF X*X
C          SIGMA       INVERSE OF SGINV
C          NWS         NUMBER OF BUSES IN WHOLE SYSTEM
C          NOS         NUMBER OF BUSES IN OWN SYSTEM
C          NIS         NUMBER OF BUSES IN INTERNAL SYSTEM
C          NBS         NUMBER OF BUSES IN BOUNDARY SYSTEM
C          NXS         NUMBER OF BUSES IN EXTERNAL SYSTEM
C          NL          NUMBER OF LINES IN WHOLE SYSTEM
C          LINOS       NUMBER OF LINES IN OWN SYSTEM
C          PCTOS       PERCENT VARIATION OF OS BUS POWERS
C          PCTXS       PERCENT VARIATION OF XS BUS POWERS
C          VX          ACCUMULATED SUM OF V*X
```

```
C         VV          ACCUMULATED SUM OF V*V
C         R           ACTUAL R
C         RHAT        ESTIMATED R
C         ZA          STORAGE FOR Z(N)
C         XA          STORAGE FOR U(N)
C         YEQIB       YEQI+YBB
C         AQX         AQX
C         YEQXL       ACTUAL YEQX+YTL
C         YTL         YTL
C         YQHAT       ESTIMATED YEQX+YTL
C         MODNU       MODNU=1 FOR BOUNDARY BUS IMPEDANCE MODEL
C                     MODNU=2 FOR TIE LINE POWER FLOW MODEL
C
C
C      SUBROUTINES
C
C         YINPT       INPUT SYSTEM PARAMETER MATRICES AND DATA
C         OUTPT       OUTPUT INFORMATION OF SIMULATION RESULTS
C         ZXV         FORM Z(N),U(N),V(N),D(N),AND PL(N) USING P(N)
C         MODEL       FORM Z,U,AND V GIVEN P AND PL
C         STATS       SUMMARIZE INFORMATION
C         PLCHK       CHECK HOW IDENTIFIED MODEL PREDICTS LINE FLOWS
C         MINVD       MATRIX INVERSION SUBROUTINE
C         RAND        RANDOM NUMBER GENERATOR
C         IMPDM       CALCULATE MATRICES USED IN SIMULATION
C
      COMMON ZWS(40,40),YL(60),KHEAD(60),KTAIL(60),P(40),PL(60),
     1D(41),PNOM(40),ABDS(5,20),YIINV(15,15),DATA(5,115),ZDSEQ(20,20),
     2Z(5),H(5,5),X(5),V(5),SIGMA(5,5),SGINV(5,5),CHI(5,5),
     3NWS,NOS,NIS,NBS,NXS,NL,LINOS,DETSG,NT,NZT,INVP,PCTDS,PCTXS
      COMMON VX(5,5),VV(5,5),BIAS(5,5),R(5,5),RHAT(5,5)
      COMMON ZA(5,800),XA(5,800)
      COMMON YEQIB(5,5),AQX(5,30),YEQXL(5,5),YTL(5),YQHAT(5,5)
      COMMON NOSG,MODNU,NTOT
      NOSG=5
C.........INPUT NETWORK MATRICES
```

```
C.........ALL INPUT IS THROUGH SUBROUTINE YINPT
      CALL YINPT
   72 CONTINUE
C.........INITIALIZE RANDOM NUMBER GENERATOR
      RR=RAND(997513.)
C.........NZT IS THE NUMBER OF MEASUREMENT SETS TO BE GENERATED
C.........IF INVP=0 SIGMA AND CHI ARE RESET TO ZERO
C.........PCTOS IS THE PERCENTAGE MOVEMENT OF OS BUSES
C.........PCTXS IS THE PERCENTAGE MOVEMENT OF XS BUSES
C.........MODNU=1 IF THE BOUNDARY BUS IMPEDANCE MODEL IS TO BE USED
C.........MODNU=2 IF THE TIE LINE POWER FLOW MODEL IS TO BE USED
   44 READ,  NZT,INVP,PCTOS,PCTXS,MODNU
      IF(PCTOS)45,41,45
C.........IF PCTOS=0, RANDOM NUMBER GENERATOR IS RESET TO VALUE IN PCTXS
   41 RR=RAND(PCTXS)
      PRINT 141,   PCTXS
  141 FORMAT (12H RAND SET TO,F14.6)
      GO TO 44
C.........IF NZT IS NEGATIVE, END PROGRAM
C.........IF NZT IS POSITIVE, GENERATE MEASUREMENTS
   45 IF(NZT)46,42,43
   43 CONTINUE
      IF(INVP)10,10,30
   10 DO 11 I=1,NBS
      DO 11 J=1,NBS
C.........VX ACCUMULATES SUM OF V*X
C.........VV ACCUMULATES SUM OF V*V
C.........CHI ACCUMULATES SUM OF Z*X
C.........SGINV ACCUMULATES SUM OF X*X
      VX(I,J)=0.0
      VV(I,J)=0.0
      CHI(I,J)=0.0
   11 SGINV(I,J)=0.0
C.........NTOT IS THE CURRENT TOTAL OF MEASUREMENTS
      NTOT=0
   30 CONTINUE
```

```
      D(NWS+1)=0.0
      NOS1=NOS+1
      NALL=3*NBS+NWS+NL
      DO 21 NT=1,NZT
C.........BEGIN GENERATION OF ONE ADDITIONAL MEASUREMENT SET
      NTOT=NTOT+1
C.........GENERATE BUS POWER CHANGES
      DO 23 I=1,NOS
   23 P(I)=PNOM(I)*PCTOS*RAND(-1.)
      DO 24 I=NOS1,NWS
   24 P(I)=PNOM(I)*PCTXS*RAND(-1.)
C.........FORM VECTORS Z,X,AND V
      CALL ZXV
C.........UPDATE STATISTICS ON Z,X,V,P,AND PL
      CALL STATS(NALL,NZT,NT)
      DO 31 I=1,NBS
      DO 31 J=1,NBS
C.........UPDATE VX,VV,CHI,AND SGINV
      VX(I,J)=VX(I,J)+V(I)*X(J)
      VV(I,J)=VV(I,J)+V(I)*V(J)
      CHI(I,J)=CHI(I,J)+Z(I)*X(J)
   31 SGINV(I,J)=SGINV(I,J)+X(I)*X(J)
      DO 25 I=1,NBS
C.........STORE Z AND X FOR LATER USE
      XA(I,NTOT)=X(I)
   25 ZA(I,NTOT)=Z(I)
C.........END GENERATION OF MEASUREMENTS
   21 CONTINUE
C.........FIND NOMINAL VALUES OF Z,X,V,P,ANDPL
      DO 20 I=1,NWS
   20 P(I)=PNOM(I)
      CALL ZXV
      CALL STATS(NALL,NZT,NZT+1)
C.........BEGIN SOLUTION FOR ESTIMATES OF H AND R
      DO 32 I=1,NBS
      DO 32 J=1,NBS
```

```
   32 SIGMA(I,J)=SGINV(I,J)
C.........FIND SIGMA=INVERSE OF SGINV
      CALL MINVD(SIGMA,NBS,DETSG,NDSG)
      IF(DETSG)40,113,40
  113 PRINT 313
  313 FORMAT (18H SGINV IS SINGULAR)
C.........FIND ESTIMATE OF H
   40 DO 48 I=1,NBS
      DO 48 J=1,NBS
      SUM=0.0
      SUM2=0.0
      DO 47 K=1,NBS
      SUM2=SUM2+VX(I,K)*SIGMA(K,J)
   47 SUM=SUM+CHI(I,K)*SIGMA(K,J)
      BIAS(I,J)=SUM2
   48 H(I,J)=SUM
      XNTOT=NTOT
C.........FIND ACTUAL R
      DO 53 I=1,NBS
      DO 53 J=1,NBS
   53 R(I,J)=VV(I,J)/XNTOT
      DO 60 I=1,NBS
      DO 60 J=1,NBS
   60 RHAT(I,J)=0.0
C.........FIND ESTIMATED R
      DO 63 K=1,NTOT
      DO 61 I=1,NBS
C.........REMOVE Z AND X FROM STORAGE
      Z(I)=ZA(I,K)
   61 X(I)=XA(I,K)
      DO 62 I=1,NBS
      DO 62 J=1,NBS
C.........FIND ESTIMATED V
   62 Z(I)=Z(I)-H(I,J)*X(J)
      DO 63 I=1,NBS
      DO 63 J=1,NBS
```

```
   63 RHAT(I,J)=RHAT(I,J)+Z(I)*Z(J)
      DO 64 I=1,NBS
      DO 64 J=1,NBS
   64 RHAT(I,J)=RHAT(I,J)/XNTOT
C..........FIND ESTIMATE OF YEQXL FROM ESTIMATE OF ZBB
      CALL MODEL(MODNU+2)
C..........OUTPUT INFORMATION
      CALL OUTPT
      GO TO 44
   42 CONTINUE
      IF(MODNU-2)71,72,72
   71 CONTINUE
C..........CALL LINE POWER CHECK SUBROUTINE TO FIND HOW WELL
C..........THE IDENTIFIED MODEL PREDICTS LINE FLOW CHANGES
      CALL PLCHK
      GO TO 44
   46 CONTINUE
      CALL EXIT
      END
```

```
C  YINPT                             3/13/71
C..........INPUT DATA TO BE USED IN SIMULATION
      SUBROUTINE YINPT
      COMMON ZWS(40,40),YL(60),KHEAD(60),KTAIL(60),P(40),PL(60),
     1D(41),PNOM(40),ABOS(5,20),YIINV(15,15),DATA(5,115),ZOSEQ(20,20),
     2Z(5),H(5,5),X(5),V(5),SIGMA(5,5),SGINV(5,5),CHI(5,5),
     3NWS,NOS,NIS,NBS,NXS,NL,LINOS,DETSG,NT,NZT,INVP,PCTOS,PCTXS
      COMMON VX(5,5),VV(5,5),BIAS(5,5),R(5,5),RHAT(5,5)
      COMMON ZA(5,800),XA(5,800)
      COMMON YEQIB(5,5),AOX(5,30),YEQXL(5,5),YTL(5),YOHAT(5,5)
      COMMON NDSG,MODNU,NTOT
C..........IF NETWORK MATRICES HAVE NOT BEEN PREVIOUSLY CALCULATED
C..........USING LINE DATA, REMOVE THE  C  IN COLUMN 1 OF THE
C..........FOLLOWING TWO CARDS
C     CALL IMPDM
C     GO TO 27
   26 CONTINUE
      READ 201,     NWS,NBS,NXS
      NOS=NWS-NXS
      NIS=NWS-NXS-NBS
      DO 21 I=1,NWS
   21 READ 202,     (ZWS(I,J),J=1,NWS)
      DO 24 I=1,NBS
   24 READ 202,   (ABOS(I,J),J=1,NOS)
      DO 25 I=1,NIS
   25 READ 202,  (YIINV(I,J),J=1,NIS)
      DO 31 I=1,NBS
   31 READ 202,    (YEQIB(I,J),J=1,NBS)
      DO 32 I=1,NBS
   32 READ 202,     (YEQXL(I,J),J=1,NBS)
      DO 33 I=1,NBS
   33 READ 202,    (AOX(I,J),J=1,NXS)
      READ 202,    (YTL(I),I=1,NBS)
      READ,  NL, LINOS
      READ,         (YL(I),KHEAD(I),KTAIL(I),I=1,NL)
   27 CONTINUE
```

```
      READ,      (PNOM(I),I=1,NWS)
  201 FORMAT (6X,I3,6X,I3,6X,I3)
  202 FORMAT (3(12X,E13.5))
      RETURN
      END
```

```
C  ZXV                                   3/16/71
C..........GENERATE Z,X,V,D,AND PL FROM BUS POWER CHANGES, P
      SUBROUTINE ZXV
      COMMON ZWS(40,40),YL(60),KHEAD(60),KTAIL(60),P(40),PL(60),
     1D(41),PNOM(40),ABOS(5,20),YIINV(15,15),DATA(5,115),ZOSEQ(20,20),
     2Z(5),H(5,5),X(5),V(5),SIGMA(5,5),SGINV(5,5),CHI(5,5),
     3NWS,NOS,NIS,NBS,NXS,NL,LINOS,DETSG,NT,NZT,INVP,PCIOS,PCTXS
      COMMON VX(5,5),VV(5,5),BIAS(5,5),R(5,5),RHAT(5,5)
      COMMON ZA(5,800),XA(5,800)
      COMMON YFOIB(5,5),AQX(5,30),YEQXL(5,5),YTL(5),YQHAT(5,5)
      COMMON NDSG,MODNU,NTOT
C..........SOLVE FOR VOLTAGE ANGLES, D
      DO 41 I=1,NWS
      D(I)=0.0
      DO 41 K=1,NWS
   41 D(I)=D(I)+ZWS(I,K)*P(K)
C..........FIND LINE POWER FLOWS, PL
      DO 42 I=1,NL
      KH=KHEAD(I)
      KT=KTAIL(I)
   42 PL(I)=YL(I)*(D(KH)-D(KT))
C..........FIND Z,X,AND V FROM P AND D
      CALL MODEL(MODNU)
C..........PLACE VALUES Z,X,V,P,AND PL IN ROW 1 OF DATA FOR USE
C..........BY SUBROUTINE STATS
      II=0
      DO 12 I=1,NBS
      II=II+1
      DATA(1,II)=Z(I)
      IB=II+NBS
      DATA(1,IB)=X(I)
      IB=IB+NBS
   12 DATA(1,IB)=V(I)
      II=3*NBS
      DO 14 I=1,NWS
      II=II+1
```

```
14 DATA(1,II)=P(I)
   DO 15 I=1,NL
   II=II+1
15 DATA(1,II)=PL(I)
   RETURN
   END
```

```
C   MODEL
C..........DEPENDING ON HOW IT IS CALLED, SUBROUTINE MODEL GENERATES
C..........Z,X,AND V FROM EITHER MODEL OR FINDS YQHAT FROM THE ESTIMATE
C..........OF H=ZBB OF THE BOUNDARY BUS IMPEDANCE MODEL
      SUBROUTINE MODEL(KPT)
      COMMON ZWS(40,40),YL(60),KHEAD(60),KTAIL(60),P(40),PL(60),
     1D(41),PNOM(40),ABOS(5,20),YIINV(15,15),DATA(5,115),ZOSEQ(20,20),
     2Z(5),H(5,5),X(5),V(5),SIGMA(5,5),SGINV(5,5),CHI(5,5),
     3NWS,NOS,NIS,NBS,NXS,NL,LINOS,DETSG,NT,NZT,INVP,PCTOS,PCTXS
      COMMON VX(5,5),VV(5,5),BIAS(5,5),R(5,5),RHAT(5,5)
      COMMON ZA(5,800),XA(5,800)
      COMMON YEQIB(5,5),AQX(5,30),YEQXL(5,5),YTL(5),YQHAT(5,5)
      COMMON NDSG,MODNU,NTOT
C..........IF KPT=1 FORM Z,X, AND V FOR THE BOUNDARY BUS IMPEDANCE MODEL
C..........IF KPT=2 FORM Z,X, AND V FOR THE TIE LINE POWER FLOW MODEL
C..........IF KPT=3 FIND YQHAT FROM ESTIMATES OF ZBB FOUND BY THE
C..........BOUNDARY BUS IMPEDANCE MODEL
C..........IF KPT=4 SET YQHAT EQUAL TO THE ESTIMATE OF H FROM THE TIE
C..........LINE PLWER FLOW MODEL
      GO TO (1,2,3,4),KPT
    1 CONTINUE
      DO 43 I=1,NBS
      INIS=I+NIS
   43 Z(I)=D(INIS)
      DO 44 I=1,NBS
      X(I)=0.0
      DO 44 J=1,NOS
   44 X(I)=X(I)+ABOS(I,J)*P(J)
      DO 46 I=1,NBS
      INBS=I+NIS
      SUM=0.0
      DO 45 J=1,NXS
      JNXS=J+NOS
   45 SUM=SUM+ZWS(INBS,JNXS)*P(JNXS)
   46 V(I)=SUM
      RETURN
```

```
    2 Z(1)=PL(17)
      Z(2)=PL(13)+PL(14)
      Z(3)=-PL(38)
      DO 51 I=1,NBS
      INIS=I+NIS
   51 X(I)=D(INIS)
      DO 52 I=1,NBS
      V(I)=0.0
      DO 52 J=1,NXS
      JNOS=J+NOS
   52 V(I)=V(I)+AQX(I,J)*P(JNOS)
      RETURN
    3 DO 61 I=1,NBS
      DO 61 J=1,NBS
   61 YQHAT(I,J)=H(I,J)
      CALL MINVD(YQHAT,NBS,DETYQ,NDSG)
      IF(DETYQ)62,113,62
  113 PRINT 213
  213 FORMAT (19H ZBBHAT IS SINGULAR)
   62 DO 63 I=1,NBS
      YQHAT(I,I)=YQHAT(I,I)+YTL(I)
      DO 63 J=1,NBS
   63 YQHAT(I,J)=YQHAT(I,J)-YEQIB(I,J)
      RETURN
    4 CONTINUE
      DO 70 I=1,NBS
      DO 70 J=1,NBS
   70 YQHAT(I,J)=H(I,J)
      RETURN
      END
```

```
C  STATS                         3/16/71
C..........EACH TIME SUBROUTINE STATS IS CALLED IT UPDATES A CONDENSED
C..........VERSION OF INFORMATION IN ROW 1 OF DATA
C..........INFORMATION KEPT INCLUDES THE RMS VALUE, THE MEAN, THE
C..........MAXIMUM VALUE AND THE MINIMUM VALUE
C..........NALL IS THE NUMBER OF COLUMNS IN DATA
C..........NEND IS THE NUMBER OF SETS OF MEASUREMENTS TO BE SUMMARIZED
C..........NN IS THE CURRENT MEASUREMENT SET NUMBER
      SUBROUTINE STATS(NALL,NEND,NN)
      COMMON ZWS(40,40),YL(60),KHEAD(60),KTAIL(60),P(40),PL(60),
     1D(41),PNOM(40),ABOS(5,20),YIINV(15,15),DATA(5,115),ZOSEQ(20,20),
     2Z(5),H(5,5),X(5),V(5),SIGMA(5,5),SGINV(5,5),CHI(5,5),
     3NWS,NDS,NIS,NBS,NXS,NL,LINOS,DETSG,NT,NZT,INVP,PCTOS,PCTXS
      COMMON VX(5,5),VV(5,5),BIAS(5,5),R(5,5),RHAT(5,5)
      COMMON ZA(5,800),XA(5,800)
      COMMON YEQIB(5,5),AQX(5,30),YEQXL(5,5),YTL(5),YQHAT(5,5)
      COMMON NDSG,MODNU,NTOT
C..........IF FIRST MEASUREMENT SET, INITIALIZE DATA
      IF(NN-1)16,16,18
   16 DO 17 I=1,NALL
      DATA(2,I)=DATA(1,I)**2
      DATA(3,I)=DATA(1,I)
      DATA(4,I)=DATA(1,I)
   17 DATA(5,I)=DATA(1,I)
      GO TO 13
   18 IF(NN-NEND)19,19,30
   19 DO 25 I=1,NALL
C..........UPDATE MINIMUM VALUES
      IF(DATA(1,I)-DATA(4,I))21,22,22
   21 DATA(4,I)=DATA(1,I)
      GO TO 24
C..........UPDATE MAXIMUM VALUES
   22 IF(DATA(1,I)-DATA(5,I))24,24,23
   23 DATA(5,I)=DATA(1,I)
C..........UPDATE RMS VALUES
   24 DATA(2,I)=DATA(2,I)+DATA(1,I)**2
```

```
C.........UPDATE MEAN VALUES
   25 DATA(3,I)=DATA(3,I)+DATA(1,I)
      GO TO 13
C.........IF NN=NEND+1 ROW 1 OF DATA HOLDS THE NOMINAL VALUES AND
C.........THE MEAN VALUES AND RMS VALUES ARE FOUND
   30 XN=NEND
      DO 31 I=1,NALL
      DATA(2,I)=SORT(DATA(2,I)/XN)
   31 DATA(3,I)=DATA(3,I)/XN
   13 CONTINUE
      RETURN
      END
```

```
C   OUTPT                              3/13/71
C.........SUBROUTINE OUTPT PRINTS ALL THE INFORMATION ACCUMULATED FOR THE
C.........LAST RUN
      SUBROUTINE OUTPT
      COMMON ZWS(40,40),YL(60),KHEAD(60),KTAIL(60),P(40),PL(60),
     1D(41),PNOM(40),ABOS(5,20),YIINV(15,15),DATA(5,115),ZOSEQ(20,20),
     2Z(5),H(5,5),X(5),V(5),SIGMA(5,5),SGINV(5,5),CHI(5,5),
     3NWS,NOS,NIS,NBS,NXS,NL,LINOS,DETSG,NT,NZT,INVP,PCTOS,PCTXS
      COMMON VX(5,5),VV(5,5),BIAS(5,5),R(5,5),RHAT(5,5)
      COMMON ZA(5,800),XA(5,800)
      COMMON YEOIB(5,5),AOX(5,30),YEQXL(5,5),YTL(5),YQHAT(5,5)
      COMMON NDSG,MODNU,NTOT
      IF(NZT)20,20,11
C.........IF NZT IS POSITIVE RESULTS OF THE IDENTIFICATION ARE PRINTED
   11 CONTINUE
      NIS1=NIS+1
      PRINT 140,  NZT,INVP,PCTOS,PCTXS
  140 FORMAT (5H1NZT=,I6,9H    INVP=,I2,10H    PCTOS=,F5.3,
     110H    PCTXS=,F5.3)
      PRINT 130
      DO 61 I=1,NBS
      PRINT 130
      PRINT 161,    (YEQXL(I,J),J=1,NBS)
   61 PRINT 162,    (YQHAT(I,J),J=1,NBS)
      PRINT 130
      PRINT 130
  161 FORMAT (10H YEQXLT    ,6F10.6)
  162 FORMAT (10H YEQXLHAT ,6F10.6)
      DO 12 I=1,NBS
      INIS=I+NIS
      PRINT 130
      PRINT 113,      (ZWS(INIS,J),J=NIS1,NOS)
      PRINT 112,      (H(I,J),J=1,NBS)
      DO 9 J=1,NBS
      JNIS=J+NIS
    9 P(J)=H(I,J)-ZWS(INIS,JNIS)
```

```
      PRINT 409,     (P(J),J=1,NBS)
      DO 10 J=1,NBS
   10 P(J)=SQRT(RHAT(I,I)*SIGMA(J,J))
   12 PRINT 410,     (P(J),J=1,NBS)
  409 FORMAT (10H ERROR     ,6F10.6)
  410 FORMAT (10H EST ERROR,6F10.6)
      PRINT 130
      PRINT 130
      DO 16 I=1,NBS
   16 PRINT 114,   (SIGMA(I,J),J=1,NBS)
      PRINT 130
      PRINT 111,   DETSG
      PRINT 130
      DO 202 I=1,NBS
  202 PRINT 312,    (SGINV(I,J),J=1,NBS)
      PRINT 130
      DO 222 I=1,NBS
  222 PRINT 315,    (CHI(I,J),J=1,NBS)
      PRINT 130
      DO 221 I=1,NBS
  221 PRINT 316,     (VX(I,J),J=1,NBS)
      II=0
      DO 57 L=1,5
      GO TO (51,52,53,54,55),L
   51 PRINT 151
      NEND=NBS
      GO TO 56
   52 PRINT 152
      NEND=NBS
      GO TO 56
   53 PRINT 153
      NEND=NBS
      GO TO 56
   54 PRINT 130
      PRINT 130
      DO 223 I=1,NBS
```

```
      PRINT 130
      PRINT 141,      (R(I,J),J=1,NBS)
  223 PRINT 142,      (RHAT(I,J),J=1,NBS)
      PRINT 131
      PRINT 154
      NEND=NWS
      GO TO 56
   55 PRINT 155
      NEND=NL
   56 PRINT 150
      DO 57 I=1,NEND
      II=II+1
   57 PRINT 157,     I,(DATA(J,II),J=1,5)
      RETURN
  130 FORMAT (1X)
  131 FORMAT (1H1)
  111 FORMAT (16H DET. OF SGINV =,E14.6)
  110 FORMAT (10H   BIAS    ,6F10.6)
  112 FORMAT (10H   HHAT    ,6F10.6)
  141 FORMAT (10H   R       ,6F10.6)
  142 FORMAT (10H   RHAT    ,6F10.6)
  113 FORMAT (10H   H       ,6F10.6)
  114 FORMAT (6H SIGMA,5F14.7)
  312 FORMAT (6H SGINV,5F14.4)
  315 FORMAT (6H CHI  ,5F14.4)
  316 FORMAT (6H VX   ,5F14.4)
  150 FORMAT (10X,7HNOMINAL,5X,
     153HRMS CHANGE    MEAN CHANGE   MIN. CHANGE   MAX. CHANGE)
  151 FORMAT (//3H  Z)
  152 FORMAT (/3H  X)
  153 FORMAT (/3H  V)
  154 FORMAT (/5H PBUS)
  155 FORMAT (/6H PLINE)
  157 FORMAT (I5,5F14.7)
C.........IF NZT IS ZERO RESULTS OF PREDICTING LINE FLOWS ARE PRINTED
   20 PRINT 131
```

```
      DO 36 I=1,NOS
      PRINT 130
      PRINT 117,  (ZWS(I,J),J=1,NOS)
      PRINT 116,  (ZOSEQ(I,J),J=1,NOS)
      DO 35 J=1,NOS
   35 P(J)=ZWS(I,J)*PNOM(J)*PCTOS
   36 PRINT 118,  (P(J),J=1,NOS)
      PRINT 131
      PRINT 120,      PCTOS,PCTXS
      PRINT 121
      PRINT 150
C..........DATA HOLDS INFORMATION ON BUS POWER CHANGES, ACTUAL LINE
C..........FLOW CHANGES, AND ERRORS IN PREDICTED LINE FLOWS
      DO 21 I=1,NWS
   21 PRINT 157,  I,(DATA(J,I),J=1,5)
      II=NWS-1
      PRINT 131
      PRINT 122
      PRINT 150
      DO 22 I=1,LINOS
      II=II+2
      PRINT 130
      PRINT 157,  I,(DATA(J,II),J=1,5)
C..........COLUMN 1 OF DATA OF DATA PRINTED HERE CONTAINS THE EFFECT OF
C..........ACTUAL NOMINAL VALUES OF PEQX ON OS LINE FLOWS
   22 PRINT 157,  I,(DATA(J,II+1),J=1,5)
      RETURN
  116 FORMAT (6H ZOSEQ,12F10.6)
  117 FORMAT (6H ZWS  ,12F10.6)
  118 FORMAT (6H Z*DP ,12F10.6)
  120 FORMAT (40H ESTIMATED PLINE CHANGES FOR OS   PCTOS=,
     1F5.3,10H     PCTXS=,F5.3)
  121 FORMAT (/5H PBUS)
  122 FORMAT (/18H PLINE OS   ACTUAL/28H PLINE OS   ESTIMATION ERROR)
      END
```

```
C   PLCHK                           3/13/71
C..........SUBROUTINE PLCHK CALCULATES THE ESTIMATED IMPEDANCE MATRIX
C..........FOR OS AND CHECKS PREDICTED LINE POWER FLOW CHANGES
      SUBROUTINE PLCHK
      COMMON ZWS(40,40),YL(60),KHEAD(60),KTAIL(60),P(40),PL(60),
     1D(41),PNOM(40),ABOS(5,20),YIINV(15,15),DATA(5,115),ZOSEQ(20,20),
     2Z(5),H(5,5),X(5),V(5),SIGMA(5,5),SGINV(5,5),CHI(5,5),
     3NWS,NOS,NIS,NBS,NXS,NL,LINOS,DETSG,NT,NZT,INVP,PCTOS,PCTXS
      COMMON VX(5,5),VV(5,5),BIAS(5,5),R(5,5),RHAT(5,5)
      COMMON ZA(5,800),XA(5,800)
      COMMON YEQIB(5,5),AQX(5,30),YEQXL(5,5),YTL(5),YOHAT(5,5)
      COMMON NDSG,MODNU,NTOT
C..........NLINT IS THE NUMBER OF BUS POWER VARIATIONS TO BE USED
      NLINT=INVP
C..........OSMV IS THE AMOUNT OS BUS POWERS ARE TO MOVED
C..........XSMV IS THE AMOUNT XS BUS POWERS ARE TO BE MOVED
      OSMV=PCTOS
      XSMV=PCTXS
      NALL=NWS+2*LINOS
      NOS1=NOS+1
      D(NWS+1)=0.0
C..........FIND ZOSEQ, ESTIMATED IMPEDANCE MATRIX FOR OS
      DO 14 I=1,NOS
      DO 14 J=1,NOS
      SUM=0.0
      DO 12 K=1,NBS
      DO 12 L=1,NBS
   12 SUM=SUM+ABOS(K,I)*H(K,L)*ABOS(L,J)
   14 ZOSEQ(I,J)=SUM
      DO 13 I=1,NIS
      DO 13 J=1,NIS
   13 ZOSEQ(I,J)=ZOSEQ(I,J)+YIINV(I,J)
      N=0
   20 N=N+1
C..........BEGIN ONE BUS POWER PATTERN VARIATION
C..........GENERATE RANDOM POWER CHANGES
```

```
      DO 61 I=1,NOS
   61 P(I)=PNOM(I)*OSMV*(RAND(0.)-0.5)*2.0
      DO 62 I=NOS1,NWS
   62 P(I)=PNOM(I)*XSMV*(RAND(0.)-0.5)*2.0
   25 DO 19 I=1,NWS
C.........STORE POWER CHANGES IN ROW 1 OF DATA
   19 DATA(1,I)=P(I)
C.........FIND ACTUAL BUS VOLTAGE ANGLES
      DO 16 I=1,NOS
      D(I)=0.0
      DO 16 J=1,NWS
   16 D(I)=D(I)+ZWS(I,J)*P(J)
      II=NWS-1
C.........FIND ACTUAL LINE POWER FLOW CHANGES IN OS
      DO 18 I=1,LINOS
      KH=KHEAD(I)
      KT=KTAIL(I)
      PL(I)=YL(I)*(D(KH)-D(KT))
      II=II+2
C.........STORE  PL IN ROW 1 OF DATA
   18 DATA(1,II)=PL(I)
C.........FIND ESTIMATED BUS VOLTAGE ANGLES
      DO 23 I=1,NOS
      SUM=0.0
      DO 22 J=1,NOS
   22 SUM=SUM+ZOSEQ(I,J)*P(J)
   23 D(I)=SUM
      II=NWS
C.........FIND PREDICTED LINE POWER FLOW CHANGES IN OS (WITHOUT USE OF PEQX)
      DO 24 I=1,LINOS
      KH=KHEAD(I)
      KT=KTAIL(I)
      II=II+2
C.........STORE IN ROW 1 OF DATA THE ERROR IN THE PREDICTED LINE FLOWS
   24 DATA(1,II)=YL(I)*(D(KH)-D(KT)) - PL(I)
C.........UPDATE STATISTICS ON P,PL, AND ERROR IN PREDICTED LINE FLOWS
```

```
      CALL STATS(NALL,NLINT,N)
      IF(N-NLINT)20,41,43
C..........FIND NOMINAL OPERATING VALUES
C..........THE PORTION OF ROW 1 OF DATA CORRESPONDING TO ERROR IN PREDICTED
C..........LINE FLOW CHANGES WILL CONTAIN THE EFFECT OF NOMINAL PEQX
C..........ON NOMINAL LINE FLOWS
   41 DO 42 I=1,NWS
   42 P(I)=PNOM(I)
      N=NLINT+1
      GO TO 25
   43 CALL OUTPT
      RETURN
      END
```

```
C   RAND      FOR 360              2/21/71
C..........RAND IS A PSEUDO RANDOM NUMBER GENERATOR WHICH IS A COMBINATION
C..........OF SUBROUTINES RANDU AND GAUSS FROM THE IBM SCIENTIFIC
C..........SUBROUTINE PACKAGE (GH20-0205-4)
      FUNCTION RAND(X)
C..........IF X IS POSITIVE, RAND IS SET TO THE VALUE OF X
C..........IF X  ZERO, A UNIFORMLY DISTRIBUTED RANDOM NUMBER BETWEEN 0.0
C..........AND 1.0 IS RETURNED IN RAND
C..........IF X IS NEGATIVE A NORMALLY DISTRIBUTED, ZERO MEAN RANDOM
C..........NUMBER WITH A VARIANCE OF 1.0 IS RETURNED IN RAND
      IF(X)3,2,1
    1 IX=X
    2 RAND=0.0
      N=1
      GO TO 4
    3 RAND=-6.0
      N=12
    4 N=N-1
      IX=IX*65539
      IF(IX)5,6,6
    5 IX=IX+2147483647+1
    6 Y=IX
      RAND=RAND+Y*.4656613E-9
      IF(N)7,7,4
    7 CONTINUE
      RETURN
      END
```

```
C   MINVD                           3/1/71
C..........MINVD INVERTS A MATRIX AND IS A MODIFIED VERSION OF
C..........SUBROUTINE MINV FROM THE IBM SCIENTIFIC SUBROUTINE
C..........PACKAGE (GH20-0205-4)
      SUBROUTINE MINVD(A,N,D,NDEM)
      DIMENSION A(NDEM,NDEM),L(50),M(50)
C         SEARCH FOR LARGEST ELEMENT
      D=1.0
      DO 80 K=1,N
      L(K)=K
      M(K)=K
      BIGA=A(K,K)
      DO 20 J=K,N
      DO 20 I=K,N
   10 IF(ABS(BIGA)-ABS(A(I,J)))15,20,20
   15 BIGA=A(I,J)
      L(K)=I
      M(K)=J
   20 CONTINUE
C         INTERCHANGE ROWS
      J=L(K)
      IF(J-K)35,35,25
   25 DO 30 I=1,N
      HOLD=-A(K,I)
      A(K,I)=A(J,I)
   30 A(J,I)=HOLD
C         INTERCHANGE COLUMNS
   35  I=M(K)
      IF(I-K)45,45,38
   38 DO 40 J=1,N
      HOLD=-A(J,K)
      A(J,K)=A(J,I)
   40 A(J,I)=HOLD
C         DIVIDE CLOUMN BY MINUS PIVOT
   45 IF(ABS(BIGA)-1.E-20)46,46,48
   46 D=0.0
```

```
      RETURN                                                            MINVD 33
   48 DO 55 I=1,N                                                       MINVD 34
      IF(I-K)50,55,50                                                   MINVD34A
   50 A(I,K)=A(I,K)/(-BIGA)                                             MINVD 35
   55 CONTINUE                                                          MINVD 36
C        REDUCE MATRIX                                                  MINVD 37
      DO 65 I=1,N                                                       MINVD 38
      HOLD=A(I,K)                                                       MINVD 39
      DO 65 J=1,N                                                       MINVD 40
      IF(I-K)60,65,60                                                   MINVD 41
   60 IF(J-K)62,65,62                                                   MINVD 42
   62 A(I,J)=HOLD*A(K,J)+A(I,J)                                         MINVD 43
   65 CONTINUE                                                          MINVD 44
C        DIVIDE ROW BY PIVOT                                            MINVD 45
      DO 75 J=1,N                                                       MINVD 46
      IF(J-K)70,75,70                                                   MINVD 47
   70 A(K,J)=A(K,J)/BIGA                                                MINVD 48
   75 CONTINUE                                                          MINVD 49
C        PRODUCT OF PIVOTS                                              MINVD 50
      D=D*BIGA                                                          MINVD 51
C        REPLACE PIVOT BY RECIPROCAL                                    MINVD 52
      A(K,K)=1.0/BIGA                                                   MINVD 53
   80 CONTINUE                                                          MINVD 54
C        FINAL ROW AND COLUMN INTERCHANGE                               MINVD 55
      K=N                                                               MINVD 56
  100 K=K-1                                                             MINVD 57
      IF(K)150,150,105                                                  MINVD 58
  105 I=L(K)                                                            MINVD 59
      IF(I-K)120,120,108                                                MINVD 60
  108 DO 110 J=1,N                                                      MINVD 61
      HOLD=A(J,K)                                                       MINVD 62
      A(J,K)=-A(J,I)                                                    MINVD
  110 A(J,I)=HOLD                                                       MINVD 64
  120 J=M(K)                                                            MINVD 65
      IF(J-K)100,100,125                                                MINVD 66
  125 DO 130 I=1,N                                                      MINVD 67
```

```
      HOLD=A(K,I)                                                       MINVD 68
      A(K,I)=-A(J,I)                                                    MINVD 69
  130 A(J,I)=HOLD                                                       MINVD 70
      GO TO 100                                                         MINVD 71
  150 RETURN                                                            MINVD 72
      END                                                               MINVD 73
```

```
C   IMPDM                           3/13/71
C..........IMPDM USES LINE ADMITTANCES AND NUMBERS OF BUSES TO WHICH
C..........LINES ARE CONNECTED TO COMPUTE ZWS,ABOS,YIINV,YEQIB,AQX,AND YTL
      SUBROUTINE IMPDM
      COMMON ZWS(40,40),YL(60),KHEAD(60),KTAIL(60),P(40),PL(60),
     1D(41),PNOM(40),ABOS(5,20),YIINV(15,15),DATA(5,115),ZOSEQ(20,20),
     2Z(5),H(5,5),X(5),V(5),SIGMA(5,5),SGINV(5,5),CHI(5,5),
     3NWS,NOS,NIS,NBS,NXS,NL,LINOS,DETSG,NT,NZT,INVP,PCTOS,PCTXS
      COMMON VX(5,5),VV(5,5),BIAS(5,5),R(5,5),RHAT(5,5)
      COMMON ZA(5,500),XA(5,500)
      COMMON YEQIB(5,5),AQX(5,30),YEQXL(5,5),YTL(5),YQHAT(5,5)
      COMMON NDSG,MODNU,NTOT
      DIMENSION YWS(40,40),ZYI(40)
      NDZW=40
      READ,      NWS,NBS,NXS
      READ,      NL,LINOS
      NOS=NWS-NXS
      NIS=NWS-NXS-NBS
      DO 11  I=1,NWS
      DO 11  J=1,NWS
   11 ZWS(I,J)=0.0
      DO 40  I=1,NL
C..........READ YL(I), THE NORMALIZED LINE ADMITTANCE, AND KHEAD(I) AND
C..........KTAIL(I), NUMBERS OF BUSES TO WHICH THE LINE IS CONNECTED
      READ,     YL(I),KHEAD(I),KTAIL(I)
      Y=YL(I)
      KH=KHEAD(I)
      KT=KTAIL(I)
      IF(KH-KT)22,22,21
   21 KA=KH
      KH=KT
      KT=KA
C..........FORM THE ADMITTANCE MATRIX AND PLACE IN ZWS TEMPORARILY
   22 ZWS(KH,KH)=ZWS(KH,KH)+Y
      IF(KT-NWS)24,24,25
   24 ZWS(KT,KT)=ZWS(KT,KT)+Y
```

```
      ZWS(KH,KT)=-Y
      ZWS(KT,KH)=-Y
   25 CONTINUE
   40 CONTINUE
C.........CALCULATE YIINV USING TEMPORARY MATRIX YWS
      DO 35 I=1,NIS
      DO 35 J=1,NIS
   35 YWS(I,J)=ZWS(I,J)
      PRINT 101
      DO 141 I=1,NIS
  141 PRINT 150,  (ZWS(I,J),J=1,NIS)
      CALL MINVD(YWS,NIS,DET,NDZW)
      IF(DET)37,36,37
   36 PRINT 136
  136 FORMAT (17H  YII IS SINGULAR)
   37 DO 38 I=1,NIS
      DO 38 J=1,NIS
   38 YIINV(I,J)=YWS(I,J)
      PRINT 100
      DO 142 I=1,NIS
  142 PRINT 150,  (YIINV(I,J),J=1,NIS)
      PRINT 100
C.........CHECK INVERSION   YII*YIINV SHOULD BE AN IDENTITY MATRIX
      DO 144 I=1,NIS
      DO 143 J=1,NIS
      ZYI(J)=0.0
      DO 143 K=1,NIS
  143 ZYI(J)=ZYI(J)+ZWS(I,K)*YIINV(K,J)
  144 PRINT 150,  (ZYI(J),J=1,NIS)
C.........CALCULATE AQI AND PLACE IN ABOS
      DO 84 I=1,NBS
      INIS=I+NIS
      DO 81 J=1,NIS
      SUM=0.0
      DO 80 K=1,NIS
   80 SUM=SUM+ZWS(INIS,K)*YIINV(K,J)
```

```
   81 ABOS(I,J)=-SUM
      DO 82 J=1,NBS
      JNIS=J+NIS
C.........ADJOIN AN IDENTITY MATRIX TO AQI MAKING ABOS
   82 ABOS(I,JNIS)=0.0
   84 ABOS(I,INIS)=1.0
      PRINT 100
      DO 145 I=1,NBS
  145 PRINT 150,  (ABOS(I,J),J=1,NOS)
C.........CALCULATE YEQIB=YEQI+YBB
      DO 91 I=1,NBS
      INIS=I+NIS
      DO 91 J=1,NBS
      JNIS=J+NIS
      YEQIB(I,J)=ZWS(INIS,JNIS)
      DO 91 K=1,NIS
   91 YEQIB(I,J)=YEQIB(I,J)+ABOS(I,K)*ZWS(K,JNIS)
      PRINT 101
      DO 42 I=1,NWS
      DO 41 J=1,NWS
   41 YWS(I,J)=ZWS(I,J)
   42 PRINT 150,  (ZWS(I,J),J=1,NWS)
      PRINT 101
C.........FIND ZWS
      CALL MINVD(ZWS,NWS,DET,NDZW)
      IF(DET)43,13,43
   43 DO 44 I=1,NWS
   44 PRINT 150,  (ZWS(I,J),J=1,NWS)
      PRINT 101
C.........CHECK INVERSION   ZWS*YWS SHOULD BE AN IDENTITY MATRIX
      DO 47 I=1,NWS
      DO 46 J=1,NWS
      SUM=0.0
      DO 45 K=1,NWS
   45 SUM=SUM+ZWS(I,K)*YWS(K,J)
   46 ZYI(J)=SUM
```

```
   47 PRINT 150,   (ZYI(J),J=1,NWS)
C.........FIND ZBB**(-1) AND PLACE IN YEQXL
      DO 92 I=1,NBS
      INIS=I+NIS
      DO 92 J=1,NBS
      JNIS=J+NIS
   92 YEQXL(I,J)=ZWS(INIS,JNIS)
      NDSG=5
      CALL MINVD(YEQXL,NBS,DET,NDSG)
      IF(DET)94,93,94
   93 PRINT 193
  193 FORMAT (16H ZBB IS SINGULAR)
C.........FIND AQX=(ZBB**(-1))*ZBX
   94 DO 95 I=1,NBS
      DO 95 J=1,NXS
      JNOS=J+NOS
      AQX(I,J)=0.0
      KNIS=K+NIS
      DO 95 K=1,NBS
   95 AQX(I,J)=AQX(I,J)+YEQXL(I,K)*ZWS(KNIS,JNOS)
C.........FIND YEQXL=YEQX+YTL = ZBB**(-1) - YEQIB + YTL
      DO 96 I=1,NBS
      DO 96 J=1,NBS
   96 YEQXL(I,J)=YEQXL(I,J)-YEQIB(I,J)
      READ,(YTL(I),I=1,NBS)
      DO 196 I=1,NBS
  196 YEQXL(I,I)=YEQXL(I,I)+YTL(I)
C.........PUNCH ALL MATRICES TO BE USED
      PUNCH 304,  NWS,NBS,NXS
      DO 75 I=1,NWS
   75 PUNCH 305,    (I,J,ZWS(I,J),J=1,NWS)
      DO 79 I=1,NBS
   79 PUNCH 309,    (I,J,ABOS(I,J),J=1,NOS)
      DO 78 I=1,NIS
   78 PUNCH 310,    (I,J,YIINV(I,J),J=1,NIS)
      DO 90 I=1,NBS
```

```
 96 PUNCH 311,    (I,J,YEQIB(I,J),J=1,NBS)
    DO 97 I=1,NBS
 97 PUNCH 312,    (I,J,YEQXL(I,J),J=1,NBS)
    DO 98 I=1,NBS
 98 PUNCH 313,  (I,J,AQX(I,J),J=1,NXS)
    PUNCH 314,    (I,I,YTL(I),I=1,NBS)
    PRINT 101
    GO TO 99
100 FORMAT (1X)
101 FORMAT (1H1)
150 FORMAT (6F12.5)
304 FORMAT (6H  NWS=,I3,6H  NBS=,I3,6H  NXS=,I3)
305 FORMAT (3(5H ZWS(,I2,1H,,I2,2H)=,F13.5))
309 FORMAT (3(5H ABO(,I2,1H,,I2,2H)=,F13.5))
310 FORMAT (3(5H YIV(,I2,1H,,I2,2H)=,E13.5))
311 FORMAT (3(5H YOB(,I2,1H,,I2,2H)=,F13.5))
312 FORMAT (3(5H YXL(,I2,1H,,I2,2H)=,F13.5))
313 FORMAT (3(5H AQX(,I2,1H,,I2,2H)=,E13.5))
314 FORMAT (3(5H YTL(,I2,1H,,I2,2H)=,F13.5))
 13 PRINT 113
113 FORMAT (17H  ZWS IS SINGULAR)
    GO TO 99
 83 PRINT 183
183 FORMAT (17H  YII IS SINGULAR)
 99 CONTINUE
    RETURN
    END
```

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